$$
\begin{gathered}
{\left[\begin{array}{ll}
745
\end{array}\right]} \\
\text { XXI. India's Contribution to licodel'sy. } \\
\text { By General J. T. Waluker, R.E., C'B., F.R.S., LL.D., late Surveyor-General of } \\
\text { India. } \\
\text { Received April 27,-Rend May 16, 1895. }
\end{gathered}
$$

[Plate 18.]

The science of Geodesy has long been indebted to India for valuable data contributing to a knowledge of the Figure of the Earth. Early in the present century the Great Trigonometrical Survey was commenced, as a basis for the general topography of the Peninsula, by Major Lambton, who found that in order to obtain exact determinations of the latitudes and longitudes of the trigonometrical points from his geodetic calculations, it was necessary to obtain more exact values of the numerical elements of the Earth's figure. He therefore commenced measuring with great care and exactitude a meridional chain of triangles, and determined the latitudes of certain stations in this chain, thereby producing a meridional arc which he carried up from Cape Comorin, in latitude $8^{\circ}$, to the parallel of $18^{\circ}$. After his death this arc was extended $11^{\circ}$ furcher to the north by Colonel Everest, who brought it up to the nearest point to the Himalayan ranges at which it seemed probable that the astronomical latitudes would not be sensibly affected by the attraction of the mountain masses. This work was completed by the year 1842, furnishing a meridional arc $21^{\circ}$ in length, which is generally known as the Great Arc of India, and has invariably been employed in all subsequent investigations of the figure of the Earth.

From that time onwards chains of principal triangulation have been carried all over India, in gridiron fashion, that is to say, by a system of meridional series at varying distances of $1^{\circ}$ to $3^{\circ}$ apart, on either side of the primary Great Arc, tied together by longitudinal series on the parallels of Madras, Bombay, and Calcutta, and along the northern frontier. Thus a large number of additional meridional chains of triangles have been furnished, several of which have been converted into meridional arcs by the addition of latitude observations at a large number of the stations. In addition to this, a large number of longitudinal arcs have been acquired by the electro-telegraphic determinations of the astronomical differences of longitude between certain of the trigonometrical stations. This work, so far as available up to the year 1878, was employed by Colonel Clarke, C.B., of the Ordnance Survey of Great Britain and

Ireland, in his latest determination of the figure of the Earth.* But very much more geodetic material has since become available.

A further contribution to geodesy has been made in the Indian pendulum observations which have been taken at several stations of the survey from Cape Comorin up to the table lands of the Himalayas, and also at islands in the ocean and points on the coast lines; they have been connected with pendulum observations in England, Russia, and America.

Full details of the principal operations of the Survey, as completed as a whole for all India, and finally reduced, are given in the " Accounts of the Operations of the Great Trigonometrical Survey," which are published in the Survey Office at Dehra Dun, in Northern India. These volumes give the measurements of the base-lines and of the principal angles, the astronomical latitudes, the electro-telegraphic longitudes, and the pendulum determinations; they also show the steps which have been taken to make the triangulation consistent throughout with the smallest possible disturbance of the facts of observation, and they give the final results which have been obtained in each instance. Of these volumes, sixteen have now been published in large quarto, the first appearing in 1870. They contain both the work which has been done by the Survey as a contribution to geodesy, as well as what has been executed as a basis for the topography of India. But they are thick, bulky volumes, and are not very readily accessible, though they have been widely distributed to public libraries. The results of the geodetic operations may, however, be summarized and capitulated in a comparatively small space; it seems desirable that this should be done, in order to place them within convenient reach of all mathematicians wishing to investigate the figure of the earth. I have therefore extracted from the published volumes all that is of essential importance in relation to geodesy, and added to it several latitude determinations which I have recently received from the Survey and which have not as yet been published, all which I trust will be found of convenience and utility in future geodetic investigations.

I now propose to give first a general sketch of the operations from their origin to their conclusion; then to describe the measurements of the base-lines and the principal angles, the instruments employed, and the method which was taken to bring the very vast amount of measurements, each one necessarily fallible, into harmony with each other, so as to secure the smallest possible disturbance of the original facts of observation. The next thing will be to give a brief description of the instruments and methods employed for the latitude observations, and a synopsis of the results, and to do the same for the electro-telegraphic determinations of differences of longitude; then to examine the extent to which these operations have been influenced by local deflections of the plumb-line on the meridian and on the prime vertical. It will be seen that great deflections of the plumb-line are met with, not only in the vicinity of mountain masses, but in plains where there are no elevations or irregularities of

[^0]surface, and where the cause of deflection must be variations of density in the masses below the ground level. This leads to the combination of the observations in groups to eliminate the influence of local attraction as far as possible. Finally, the most probable values of the amplitudes, and the lengths in feet of certain meridional and longitudinal ares resulting from these combinations, are given.

## General Sketch of the Operations.

The Great Trigonometrical Survey of India was commenced in the year 1800, when a systematic triangulation was initiated in the Southern Peninsula, by Major Lambтon, with a view to determining the exact positions in latitude and longitude of all the objects that appeared best calculated to become permanent geographical marks to be employed as the basis of the general survey of the country. Up to that time whatever surveys had been made had depended on astronomical determinations of latitudes and longitudes, which were liable-the latter more particularly-to far greater errors than the differential determinations of the same elements which might be obtained from a good triangulation. The Astronomical Observatory at Madras had then been founded, and Major Lambton made it the origin of his longitudes, through which to refer to the meridian of Greenwich.

But the accuracy of geodetic calculations of latitude and longitude from the data of a triangulation depends on the accuracy of the elements of the figure of the earth which are employed in the calculations. At that time those elements were not known with all desirable accuracy, for though geodetic operations had been completed in various parts of the world, they had not yet been undertaken in regions within $33^{\circ}$ of the equator. Thus Major Lambton executed a central meridional triangulation with the greatest possible precision, and measured the astronomical latitudes of some of the stations, thereby initiating the Indian Great Arc as a contribution to the science of geodesy, and a means of enhancing the accuracy of his own operations. He carried this arc from Cape Comorin to the parallel of $18^{\circ}$, and made a general triangulation of the hill country to the east and west up to the coast lines, founding it on chainmeasured base-lines which were placed at convenient distances apart.

He was succeeded by Colonel Everest, who carried the Great Arc up to the Himalayan Mountains, and made many notable improvements in the methods of operation. He introduced luminous signals-heliotropes for observations by day, and lamps with parabolic reflectors for observations by night-which were much more satisfactory than the opaque signals hitherto employed, and which could only be observed during the day-time. He obtained better theodolites for measuring the principal angles, and Colby's apparatus of compensation bars and microscopes for measuring base-lines in place of the measuring chains which had hitherto been employed. He also made an important modification in the general design of the principal triangulation, which has resulted in greatly increasing its value for geodetic
purposes; instead of throwing it over the whole country from coast to coast as had been done in southern India, he adopted the gridiron form of meridional chains tied together by longitudinal chains, the normal directions to be only deviated from on the frontier and coast lines, the directions of which had necessarily to be followed.

Colonel Everest retired in 1843, and was succeeded by Colonel Waugr, under whose superintendence a large amount of the principal triangulation was executed. Additional great theodolites were obtained, the precision of the modus operandi was increased by further refinements, and some of the new chains of triangles became superior in accuracy to the Great Arc. Thus they would become available for meridional and longitudiual arcs for geodetic purposes by the addition of the determination of astronomical latitudes and differential-longitudes, and Colonel Waugh took steps to obtain new instruments from England for the purpose.

Colonel Waugh retired in 1861, and was succeeded in the superintendence of the Great Trigonometrical Survey by the author of this paper, under whom the principal triangulation was completed in India proper and carried down through Burma to Mergui, and considerable progress was made with the astronomical latitudes and differential-longitudes. He retired in 1884, after which the astronomical work was carried on under the superintendence of Colonel George Strahan.

It should be here stated that in order to place all the principal triangulation on a satisfactorily uniform basis of accuracy, Major Lambton's portion of the Great Arc has been wholly revised, and the gridiron system has been carried down the east and west coasts of the Peninsula to the parallel of Madras, and down the east coast to Cape Comorin, with a branch series across the Straits of Manaar, to form a connection with the triangulation of the Island of Ceylon. The annexed chart (Plate 18) shows the whole of the principal triangulation which has been executed up to date, and also the secondary triangulation outside the limits of the principal ; a vast amount of secondary work has also been executed within the limits of the principal as a basis for topographical surveys; but all this is omitted, as it would only confuse the chart.

I now proceed to give sufficient information regarding the principal triangulation and the base-lines on which it rests, and the method by which the whole has been finally reduced and rendered consistent all over India, to indicate the value of the final results and the amount of confidence which may be placed on them for future geodetic investigations.

## The Base-lines.

The base-lines are ten in number; half of them are situated on the Great Are, at Cape Comorin, Bangalore, Bider, Sironj, and Dehra Dun, and the remainder at Attok, Kurrachee, Sonakhoda, Calcutta, and Vizagaparam, which are situated at the further corners of certain of the gridiron figures which have been made in combination with various sections of the Great Arc. They were all measured with Colbr's apparatus of compensation bars and microscopes, which is described in detail in vol. 1 of the ' Account of the Operations,' and in many other publications.

Though the base-line measurements were invariably carried on under tents, for the protection of the bars and microscopes from the rays of the sun, it was soon found that the lengths of the compound bars did not remain constant throughout the day ; they were consequently compared systematically with the standard 10 -foot iron bar of the Survey for three days before and after the measurement of each base-line, at the same hours as the actual measurements, and their lengths were assigned accordingly. It was at first supposed that the compensation points had not been exactly fixed by the makers; but eventually it was found at the Cape Comorin base-line-where a line of about one-fourth the usual length was measured over four times, in order to test the apparatus-that whichever of the two components of a bar was nearest the sun acquired heat most rapidly, notwithstanding that they lay side by side in a stout deal case, and were further protected by tents. The variations in length, however, though very sensible under high microscopic power, were not of a material magnitude; and as in the comparisons of the compound bars with the standard which were made at each base-line, the bars were invariably set up parallel to the line of the measurement and compared during the hours in which the measurements were made, the errors arising from the variations in length must have been largely eliminated.

The mathematical probable errors of the base-lines are investigated in chapters ix. and $x$. of vol. 1 , where it is shown that all the ten base-lines may be practically regarded as of equal weight, and that their probable errors may be taken as equal to $\pm 2.6$ millionth parts of the length measured, which corresponds to 108 feet in the length of the polar axis of the earth.

## The Principal Triangulation.

The principal angles were all measured with theodolites whose azimuthal circles ranged from 36 inches down to 15 inches in diameter, the larger circles being furnished with five and the smaller with three micrometer microscopes for reading the graduations. More than three-fourths of the angles were measured with the larger theodolites having circles of 24 or 36 inches in diameter. All the instruments are described in Appendix No. 2 of vol. 2 of the 'Account of the Operations.'

The signals between which the angles were measured were invariably luminous, heliotropes by day, and lamps by night.

The system of operation was specially devised with a view to eliminate the graduation errors of the azimuthal circles to the utmost possible extent, by systematic angular shifts in the position of the zero of the circle in the course of the successive measurements of each angle, so that the microscope readings during the telescope pointings to any object might be taken at several equidistant points on the circle. The distances were taken first at $9^{\circ}$, and atterwards at $7^{\circ} 12^{\prime}$, for circles with five microscopes, and at $10^{\circ}$ for circles with three microscopes, producing a greater number of changes of zero for the latter than for the former. Thus in the final mean result, 5 D 2
periodic errors of graduation are eliminated, and accidental errors are largely cancelled because of the large number of graduations employed. Half the measurements were made with the face of the vertical circle to the right, and half with it to the left of the observer, thus cancelling the effects of collimation error, and doubling the number of points at which the circle was read, the number of microscopes being always an odd number. Sometimes two, but more generally three measures were taken as a minimum in each position of the instrument, before changing face or altering the zero setting. The minimum number prescribed for the whole of the measures varied with the adopted ares for zero setting, and was usually 20,24 , or 30 for theodolites having five microscopes, and 36 for those having three; its completion was an invariable condition of observation, and it was always added to whenever unfavourable atmospheric conditions made it difficult to intersect the signals accurately.

The observations completed, the weight of each angle was deduced as usual from the discrepancies between each measure and the mean of all. For all the later triangulation this has been done by a formula which endeavours to take account of both the personal error of observation, as shown by the differences between measures taken on the same graduations, and the graduation error, as shown by the differences between each zero mean and the mean of all the zeros. The theoretical errors of mean square of observation and of graduation are given in the 'Accounts of Operations,' immediately after the details of the observations of the angles in each chain of triangles, and they are combined in separate groups for each observer and instrument, and also for observations at stations on hills and at stations on plains, as the former are usually taken under more favourable atmospheric conditions. The theoretical errors are given only for single observations and for single zeros, for which they are mathematically exact; but the theoretical error of the value of the angle deduced from the mean of all the measures, which must necessarily be deduced from them eventually, is only exact when the graduation errors are all accidental, for periodic errors of graduation, however large, are wholly eliminated by the systematic changes of zero. When they exist the theoretical errors will be too large. This does not matter for the primary reduction of triangles whose angles have all been measured with the same instrument; but it is of importance when masses of triangulation with different instruments have to be put together and finally adjusted. Thus it eventually became necessary to correct the preliminary weights by the more reliable evidence of average weight, which can be obtained from an examination of the geometrical errors of triangles and polygonal figures, of which a large number have been measured with the same instrument.

The following table gives the average values of the theoretical probable errors of the angles measured by the several theodolites, which were derived from the actual observations. The number of angles and their average probable errors are given for each of the separate sections into which the triangulation was divided for final reduction, as will presently be explained.

GENERAL J. T. WALKER ON INDIA'S CONTRIBUTIUN TO GEODESY.

| Theodolite. | North-West Quadrilateral. |  | North-East Quadrilateral. |  | South-East Quadrilateral. |  | South-West Quadrilateral. |  | Trigon. |  | Whole. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. | Probable error. | No. | Probable error. | No. | Probable error. | No. | Probable error. | No. | Probable error. | No. | Probable error. |
| Troughton and Simms' 36 -inch . | 1104 | $\pm \cdot 2 l$ | 35 | $\pm \cdot 29$ | $560^{\circ}$ | $\pm \cdot 20$ | - | ". | 4.5 | $\begin{array}{r} \prime \prime \\ \pm \quad 17 \end{array}$ | 1750 | + ${ }^{\prime} \cdot 21$ |
| Barrow's 36-inch | 164 | -32 | 156 | $\cdot 19$ | 66 | -23 | - | . | $\cdots$ | - | 386 | $\cdot 25$ |
| Troughton and Simas' 24-inch, No. 1 | 57 | -19 | . | . | 211 | $\cdot 27$ | - | -• | 946 | $\cdot 16$ | 1214 | -18 |
| Troughton and Simms' 24-inch, No. 2 | 147 | $\cdot 21$ | 395 | $\cdot 20$ | -• | . | - | . | $\cdots$ | -• | 542 | $\cdot 20$ |
| Barrow's 24-inch, No. I . | $\cdots$ | . | 465 | $\cdot 35$ | $\cdots$ | . | . | - | $\cdots$ | . | 465 | -35 |
| Barrow's 24-inch, No. 2. | 373 | $\cdot 14$ | 48 | -40 | 99 | $\cdot 33$ | -• | - | 489 | $\cdot 21$ | 1309 | -16 |
| Waugh's 24-inch, No. 1 | 638 | $\cdot 24$ | 123 | $\cdot 37$ | 391 | $\cdot 21$ | $\cdots$ | -• | $\cdots$ | . | 1152 | $\cdot 24$ |
| Waugh's 24-inch, No. 2 | 251 | $\cdot 35$ | 33 | $\cdot 35$ | . | . | . | . | $\cdots$ | - | 284 | $\cdot 35$ |
| Trovghton and Simms' 18-inch, No. 1 | -81 | 1.95 | 276 | 1.55 | 13 | $1 \cdot 47$ | $\cdots$ | . | - | - | 370 | 1.63 |
| Troughton and Simms' 18-inch, No. 2 | - | - | 209 | . 60 | -• | $\cdots$ | 698 | $\pm 61$ | - | -• | 907 | $\cdot 61$ |
| Cary's 18-inch | . | -• | 89 | -79 | -• | - | $\cdots$ | -• | $\cdots$ | $\cdots$ | 89 | -79 |
| Stod Mohsin's 18-inch | . | . | 78 | . 93 | . | -• | $\cdots$ | $\cdots$ | -• | - | 78 | $\cdot 93$ |
| Harris and Barrow's 18-inch . | . | . | 195 | -69 | . | - | $\cdots$ | . | -• | -• | 195 | $\cdot 69$ |
| Cary's 15-inch . : | - | $\cdots$ | 147 | $\cdot 86$ |  | $\cdots$ | $\cdots$ | -• | $\cdots$ | - | 147 | -86 |
| Dollond's 15-inch . | 43 | . 83 | . | . | . | . | 190 | . 88 | 125 | .91 | 358 | . 88 |

The values of the probable error which were obtained from an examination of the triangular and polygonal geometrical errors give results which slightly exceed the above values, excepting in the case of Troughton and Simms' 18 -inch theodolite, No. 1 ; this instrument has been found to have a large periodic error, which becomes eliminated by the adopted method of changing zero, as is shown in Appendix No. 4, vol. 2 ; its true theoretical probable error is only about one-third of the quantity shown in the preceding table.*

The several series of triangles are composed sometimes of single triangles, but most frequently of polygonal figures, which are self-verificatory, and add considerable weight to the triangulation. The single triangles mainly appertain to the portions of the triangulation lying wholly in the plains which were first completed, but, as the work proceeded, polygonal figures were employed in plains as well as in hill districts.

During the recess following each field season, the reduction of the observations was taken in hand, the weight of each angle was determined, the corrections to the observed angles, which were necessary to satisfy the geometrical conditions involved, were computed by the method of minimum squares, the lengths and azimuths of the sides, and the latitudes and longitudes and heights of the stations were calculated; the secondary triangulation was treated somewhat similarly, but with less precision, and thus all the work of the previous field-season was rendered available for immediate topographical requirements.

But it will be obvious that as the several chains of triangles became completed, closing on each other and on the base lines, discrepancies would be met with in the values of length, azimuth, latitude, and longitude, which would be presented by the different chains at the sides of junction. In order to make the triangulation consistent throughout, it was necessary to eliminate these discrepancies by the application of small corrections to the whole of the angles, and it became a matter of great importance to decide how this should be done. To do it with strict theoretical nicety would necessitate the simultaneous reduction of the whole of the triangulation which would

[^1]have entailed the solution of an algebraical problem, presenting upwards of 8000 unknown quantities, connected together by upwards of 3600 geometrical equations of condition, and the operation could not have been commenced until the whole of the triangulation was completed. Strict theoretical nicety was, therefore, set aside. The triangulation, as a whole, was divided into five great sections, to be treated succes. sively, on the understanding that the whole of the angular corrections for each section should be computed simultaneously, but the angles of chains of triangles, which might be common to any two sections, should retain the corrections obtained for them in the section first reduced, and should not be introduced into the reduction of the second section.

The five sections thus decided on for separate treatment comprise four quadrilateral figures for Northern and Central India and a trigon for the Southern Peninsula. The northern sections lie on either side of the Great Arc and meet at the station of Kalianpur, in Central India, which Colonel Everest adopted as the origin of the geodetic calculations of the survey; they are respectively called the North-East Quadrilateral, the North-West Quadrilateral, the South-East Quadrilateral, and the South-West Quadrilateral ; the fifth section is called the Southern Trigon, and is bisected by the Great Arc.

In the simultaneous solutions of the several sections much simplification was rendered possible by the circumstance that, when a large number of independent equations has to be treated, we may select and solve any groups we please, independently, and apply corrections to the angles as deduced without prejudice to obtaining the most probable values of the angles ultimately, provided the whole of the equations are finally solved simultaneously with such absolute terms as exist after the approximations have been made. An important first approximation had already been made by the correction of the observed values of the angles for the genmetrical errors of the triangles and polygonal figures to which they respectively appertained; thus, each polygonal triangulation might be converted into a chain of single triangles by employing the triangles on one flank of the polygons only and discarding those on the other flank, and, as the angles had already been made consistent, it did not matter which flank was employed. After the final values of the angles on the adopted flank had been obtained by the simultaneous solution, those for the discarded angles could be readily obtained, figure by figure, and thus the whole triangulation would be finally adjusted.

It is here desirable to observe that the numerous astronomical determinations of azimuth, latitude, and differential longitude which have been made at various stations of the Survey, furnish a number of external and independent facts of observation with which the triangulation might have been brought into accordance. The azimuths more particularly seemed to have a claim for introduction, as they were measured with the intention of forming a check on the triangulation; they are very numerous, and are given in full detail in the accounts of the operations. But the triangulation
of this Survey cannot be forced into accordance with any astronomically determined facts of observation without a liability to introduce errors, arising from deflections of the plumb-line in astronomical observations, which far exceed the errors ordinarily generated in the triangulation. Moreover, other errors would arise, more minute, but becoming very sensible in long distances, from the errors in the numerical elements of the earth's figure which were adopted as the basis of the geodetic calculations of latitude, longitude, and azimuth. If the triangulation were to be forced into accordance with the astronomical determinations, it would become seriously distorted in parts and unfitted for employment in further investigations of the figure of the Earth. Thus, in the final reductions, the lengths of the base-lines and the geometrical consistency of the angles were alone taken into consideration, the only astronomically determined elements which were employed being those which were adopted for Kalianpur, the station of origin of the calculations.

As each section would have to accept the results of the reduction of a previous section as final and unalterable, it was obviously necessary to commence the final reductions with the section containing the most accurately measured angles and follow with the next most accurate section, in order that work of the best class might not be injured by being forced into accordance with work of an inferior nature. The reductions were commenced about the year 1868, when the NorthWest Quadrilateral was completed, with the exception of two comparatively short chains across the desert tracts of Sind and Rajputana, which were postponed until the whole of the rest of the triangulation had been completed. The North-East Quadrilateral was already completed; but it consists of a number of series of triangles which were the first executed after the completion of the Great Arc, with less precision than the work of subsequent years, and several of them with the inferior 18 -inch and 15 -inch theodolites; and there was reason to fear that portions of the triangulation which were carried on towers over the plains of Bengal had suffered from slight displacement of the markstones by deflections of the towers during the heavy rains of the Indian monsoons, thus altering the lengths of certain sides by a few inches in the interval between closing and resuming operations.* The North-West Quadrilateral was of much superior accuracy to the North-East Quadrilateral; it was therefore determined to commence the final reductions with that section as already completed, leaving the two intermediate chains to be afterwards fitted in. By the time its reduction was completed the triangulation of the SouthEast Quadrilateral was also completed, and its final reduction was taken in hand next for similar reasons.

Thus the order adopted for the final simultaneous reductions of the several sections was as follows:-

[^2]First Section.-The North-West Quadrilateral. Second Section.-The South-East Quadrilateral. Third Section.-The North-East Quadrilateral. Fourth Section.-The Southern Trigon. Fifth Section. -The South-West Quadrilateral.

Full details of the measurements of the observed angles and of the successive steps of the reductions and the final results are given in vols. 2, 3, and 4 for the first section, vol. 6 for the second, vols. 7 and 8 for the third, vols. 12 and 13 for the fourth, and vol. 14 for the fifth.
The labour involved in each of the great simultaneous reductions was very considerable; the mass of calculation was enormous, and as, in each case, it was all interdependent, so that a single error might vitiate the results, great precautions had to be taken to eliminate error in each stage of the computations before proceeding to the next stage. A description of the practical procedure in the case of the NorthWest Quadrilateral is given by Mr. Hennessey in Appendix No. 12 to vol. 2. Some modifications and improvements were introduced in the treatment of the subsequent sections by Colonel Herschel and Mr. W. H. Cole. But in every instance the reduction was a very anxious as well as laborious operation, and its successful completion was a matter of much gratulation and satisfaction.
'The numerical values of the several discrepancies in length, azimuth, latitude, and longitude, at the sides of junction of the chains of triangles and at the base-lines, which were met with in each of the sections, and adjusted by the processes of simultaneous reduction, will now be exhibited. Afterwards the magnitudes of the corrections to the angles which resulted from the calculations will be set forth.

## The North-West Quadrilateral.

The outlines of this figure, as completed up to the time when its final reduction was taken in hand, are given in this sketch.


It is composed of eight chains of triangles, resting on four base-lines, one at each of the corners of the quadrilateral, viz., at Sironj, Dehra Dun, Attok, and Kurrachee.

The chains are-

> A, the Great Arc, section $24^{\circ}$ to $30^{\circ}$. B, the Kurrachee Longitudinal Series in three sections, $\mathrm{B}_{1}, \mathrm{~B}_{2}$, and $\mathrm{B}_{3}$. C, the North-West Himalaya Series in four sections, $\mathrm{C}_{1}$ to $\mathrm{C}_{4}$. D, the Great Indus Series in two sections, $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$. E, the Rahun Meridional Series.
> F, the Gurhagarh Meridional Series in two sections, $\mathrm{F}_{1}$ and $\mathrm{F}_{2}$. G, the Jogi-Tila Meridional Series.
> H, the Sutlej Series in two sections, $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$.

The number of angular errors for simultaneous determination was 1650 , which were connected together by 23 equations of condition, some comprising the errors in the direct triangulations between the base lines, and the remainder comprising the errors in the circuits of triangulation. The formation of these equations is to some extent a matter of choice; in the present instance all the linear or side-length equations were first written down, and afterwards the equations to the geodetic errors on the circuits; but eventually it was found preferable, whenever possible, to place the linear and the three geodetic equations appertaining to each circuit together, as this causes less entanglement and labour of calculation in the solution.

In the following table the errors of the logarithms of the side-lengths in feet, and the geodetic errors in seconds of arc, are shown. The column for included triangles shows the number of single triangles forming the chain or circuit and omits the discarded angles of the polygonal figures.

| Chains and their sections. | Included triangles. | Errors. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Log. side-length. | Latitude. | Longitude. | Azimuth |
| A | 51 | + $\cdot 000,0044,0$ | " | " | " |
| $\mathrm{B}=\mathrm{B}_{1}+\mathrm{B}_{2}+\mathrm{B}_{3}$. | 96 | -.000,0079,6 |  |  |  |
| $\mathrm{C}=\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}+\mathrm{C}_{4}$ | 39 | +.000,0071,9 |  |  |  |
| $\mathrm{D}=\mathrm{D}_{1}+\mathrm{D}_{2}$. | 123 | + $000,0163,8$ |  |  |  |
| $\mathrm{C}_{1}-\left(\mathrm{B}_{1}+\mathrm{E}\right)$. ${ }^{\text {d }}$ | 94 | + $000,0068,2$ |  |  |  |
| $A+C_{1}-\left(B_{1}+E\right) \cdot \dot{ }$ | 145 |  | +0.391 | $+0 \cdot 168$ | +5.91 |
| $\mathrm{E}+\mathrm{C}_{2}-\left(\mathrm{B}_{2}+\mathrm{F}_{1}+\mathrm{F}_{2}\right)$ | 185 | - 000,0124,6 | -0.390 | +0.214 | +1.55 |
| $\mathrm{F}_{2}+\mathrm{C}_{3}-\left(\mathrm{H}_{1}-\mathrm{G}\right)$. | 87 | + $0000,0150,9$ | $+0.036$ | $-0.287$ | -4.23 |
| $\mathrm{G}+\mathrm{C}_{4}-\left(\mathrm{H}_{2}+\mathrm{D}_{2}\right)$. $\mathrm{F}_{1}+\mathrm{H}_{1}+\mathrm{H}_{2}-\left(\mathrm{B}_{3}+\mathrm{D}_{1}\right)$. | 139 250 | -.000,0005,3 | -0.005 +0.387 | -0.290 +0.293 | -3.00 -3.25 |
| $\mathrm{F}_{1}+\mathrm{H}_{1}+\mathrm{H}_{2}-\left(\mathrm{B}_{3}+\mathrm{D}_{1}\right)$ | 250 | . | +0.387 | +0.293 | -3.25 |

## The South-East Quadrilateral.

This quadrilateral is composed of six chains of triangles, and its corners rest on the base-lines at Sironj, Bider, Vizagapatam, and Calcutta. It is wholly executed with

the best instruments, two small meridional chains near Calcutta, which were executed with inferior instruments, having been excluded from the final simultaneous reduction. The chains employed are-

A, the Great Arc, section $18^{\circ}$ to $24^{\circ}$.
B, the Calcutta Longitudinal Series in three sections, $B_{1}, B_{2}$, and $B_{3}$.
C, the Coast Series.
D, the Bider Longitudinal Series in three sections, $\mathrm{D}_{1}, \mathrm{D}_{2}$, and $\mathrm{D}_{3}$.
E, the Jabalpur Meridional Series.
F, the Bilaspur Meridional Series.
The number of angular errors involved was 831, which were included in fifteen equations of condition, six linear and nine geodetic, presenting the following errors:-

| Chains and their sections. | Included triangles. | Errors. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Log. side-length. | Latitude. | Longitude. | Azimath. |
| A | 40 | - $\cdot 000,0016,4$ | " | " | " |
| $\mathrm{D}_{1}-\left(\mathrm{B}_{1}+\mathrm{E}\right)$ | 65 | - $\cdot 000,0054,9$ |  |  |  |
| $\mathrm{C}^{\text {c }}$ - | 65 | - $\cdot 000,0006,9$ |  |  |  |
| $B=B_{1}+B_{2}+B_{3}$ | 54 | + 00000042,5 |  |  |  |
| $D=D_{1}+D_{2}+D_{3}$ | 39 | - 000,0021,3 |  |  |  |
| $\mathrm{A}+\mathrm{D}_{1}-\left(\mathrm{B}_{1}+\mathrm{E}\right)$ | 100 |  | +0.050 | -0.214 | +0.21 |
| $\mathrm{E}+\mathrm{D}_{2}-\left(\mathrm{B}_{2}+\mathrm{F}\right)$ | 109 | +-000,0031,9 | -0.216 | +0.203 | -4.97 |
| $\mathrm{F}+\mathrm{D}_{3}-\left(\mathrm{B}_{3}+\mathrm{C}\right) . . .$. | 154 | +00,0031,9 | -0.142 | $+0.239$ | $-3.89$ |

## North-East Quadrilateral.

This Quadrilateral includes the whole of the series of triangles east of the Great Arc and north of the parallel of Calcutta. The reduction of this figure rests on the final results already obtained, for the section of the Great Arc to the west and for the Calcutta Longitudinal Series to the south, from the reductions of the NorthWest and the South-East Quadrilaterals ; it also rests on the Sona-Khoda base-line which is situated at the northern extremity of the Calcutta Meridional Series. It very clusely touches the base-lines at Calcutta, Sironj, and Dehra Dun, and rests on the polygonal figures of which they form a part.
The outlines of the figure are as follow :-


Thus the angles which came under treatment were those of the following series:-
I, the North-East Longitudinal Series, in ten sections, $I_{1}$ to $I_{10}$.
K, the Rangir Meridional Series.
L, the Amua Meridional Series.
M, the Karára Meridional Series.
N , the Gurwani Meridional Series.
O, the Gora Meridional Series.
P, the Hurilaong Meridional Series.
Q, the Chendwar Meridional Series.
R, the North Parasnath Meridional Series.
S, the North Maluncha Meridional Series.
T, the Calcutta Meridional Series.
U, the East Calcutta Longitudinal Series, in two sections, $\mathrm{U}_{1}$ and $\mathrm{U}_{8}$.
V, the Brahmaputra Meridional Series.
W, the Eastern Frontier Series, section $23^{\circ}$ to $26^{\circ}$.
X , the Assam Longitudinal Series, in two sections, $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$.
The number of angular errors for determination was 1719, and the number of equations of condition was 49

The errors to be eliminated were as follow :-

| Chains and their sections. | Included triangles. | Errors. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Log. side-length. | Latitude. | Longitude. | Azimuth. |
| $\mathrm{K}+\mathrm{I}_{1}$ | 44 | + $\cdot 000,0287,1$ | +0"939 | +0.'934 | +11'60 |
| $\underline{L}+\mathrm{I}_{2}-\mathrm{K}$ | 82 | -.000,0007,3 | -0.352 | $-1.090$ | -14.32 |
| $\mathrm{M}+\mathrm{I}_{3}-\mathrm{L}$ | 81 | + 000,0482,3 | +0.925 | -0.183 | -6.80 |
| $\mathrm{N}+\mathrm{I}_{4}-\mathrm{M}$ | 79 | - 000,0641,0 | $-1.209$ | -0.060 | +10.18 |
| $\mathrm{O}+\mathrm{I}_{5}-\mathrm{N}$ | 73 | +-000,0169,4 | +0.383 | +0.237 | - 0.79 |
| $\mathrm{P}+\mathrm{I}_{6}-\mathrm{O}$ | 73 | + 000,0082,1 | +0.211 | +0.104 | + 4.84 |
| $\mathrm{Q}+\mathrm{I}_{7}-\mathrm{P}$ | 69 | - 000,0132,9 | -0.357 | -0.030 | - $5 \cdot 74$ |
| $\mathrm{R}+\mathrm{I}_{8}-\mathrm{Q}$ | 55 | + 000,0196,7 | +0.303 | -0.027 | + 4.44 |
| $\mathrm{S}+\mathrm{I}_{9}-\mathrm{R}$ | 52 | --000,0284,6 | -0.314 | +0.036 | - 2.28 |
| $\mathrm{T}+\mathrm{I}_{10}-\mathrm{S}$ | 83 |  | -0.214 | -0.113 | + 0.65 |
| $\mathrm{I}_{10}+\mathrm{S}$ | 36 | + $\cdot 000,0190,9$ |  |  |  |
|  | 47 | + $\cdot 000,0036,7$ |  |  |  |
| $\mathrm{U}_{1}+\mathrm{V}-\mathrm{X}_{1}$. | 81 | -.000,0193,7 |  |  |  |
| $\mathrm{U}_{1}+\mathrm{V}-\left(\mathrm{T}+\mathrm{X}_{1}\right)$ | 128 |  | +0.129 | -0.143 | + 2.41 |
| $\mathrm{U}_{9}+\mathrm{W}-\left(\mathrm{V}+\mathrm{X}_{\mathbf{2}}\right)$ | 113 | + $\cdot 000,0102,0$ | +0.173 | -0.627 | -13.14 |

## The Southern Trigon.

The triangulation rests on the Bider Longitudinal Series of the South-East Quadrilateral and on two of the base-lines of the Great Arc, one at Bangalore, the other at Cape Comorin. Its outlines are shown in the following sketch :-


It is composed of seven chains of triangles-
A, the Great Arc from $8^{\circ}$ to $18^{\circ}$, in three sections, $\mathrm{A}_{1}, \mathrm{~A}_{2}$, and $\mathrm{A}_{3}$.
$B$, the Bombay Longitudinal Series, in two sections, $B_{1}$ and $B_{2}$.
C, the South Konkan Coast Series.
D, the Mangalore Meridional Series, in two sections, $D_{1}$ and $D_{2}$.
$E$, the Madras Meridional and Coast Series, in three sections, $\mathbf{E}_{1}, \mathbf{E}_{2}$, and $\mathbf{E}_{3}$. F, the South-East Coast Series.
$G$, the Madras Longitudinal Series, in two sections, $G_{1}$ and $G_{2}$.
The number of angular errors involved was 909 , and they were embodied in 22 equations of condition.

The errors to be eliminated were as follows :-

| Chains and their sections. | Included triangles. | Errors. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Log. side-length. | Latitude. | Longitude. | Azimuth. |
| $\mathrm{B}_{1}+\mathrm{C}-\mathrm{D}_{1} \cdot .^{\text {. }}$. | 46 | + $\cdot 000,0136,6$ | +0.022 | +0.078 | -0゙125 |
| $B_{2}+D_{1}+D_{2}+G_{1}-A_{1}$ | 99 | - $000,0022,7$ | -0.001 | +0.288 | -3.68 |
| $\mathrm{A}_{1} \cdot \cdot \cdot \cdot{ }^{\text {a }}$. | 35 | + 000,0020,0 |  |  |  |
| $A_{1}+G_{2}-\left(E_{1}+E_{2}\right)$ | 84 | + 000,0039,9 | $0 \cdot 000$ | -0.174 | $+4.30$ |
| $\mathrm{G}_{2}+\mathrm{F}+\mathrm{A}_{8}$ $\mathrm{~A}_{2}-\left(\mathrm{G}_{2}+\mathrm{F}\right)$. | 88 107 | + $0000,0021,5$ |  |  |  |
| $A_{0}-\left(G_{2}+F\right)$ $A_{2}+A_{3}$. | 107 33 |  | -0.273 | +0.424 | -9.04 |
| $\mathrm{A}_{2}+\mathrm{A}_{3}$ $\mathrm{E}_{1}-\mathrm{E}_{3}$. | 33 32 | $\begin{aligned} & +\cdot 000,0001,1 \\ & -\cdot 000,0011,7 \end{aligned}$ | -0.042 | -0.001 | -0.32 |

The South-West Quadrilateral.
This triangulation rests on the North-West Quadrilateral and on the Southern Trigon. The number of angular errors for determination was 516 , and the number of equations of condition 24. Its outlines are shown in the following sketch :-


It consists of six chains of triangles, viz. -
$G$, the Khanpisura Meridional Series, in two sections, $G_{1}$ and $G_{2}$. H , the Singi Meridional Series, in two sections, $\mathrm{H}_{1}$ and $\mathrm{H}_{2}$. I, the Abu Meridional Series.
$J$, the Kattywar Meridional Series, in two sections, $J_{1}$ and $J_{8}$. $K$, the Guzerat Longitudinal Series, in three sections, $K_{1}, K_{2}$, and $K_{3}$. L, the Cutch Coast Series.

The errors to be eliminated were as follows :-

| Chains and their sections. | Included triangles. | Errors. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Log. side-length. | Latitude. | Longitude. | Azimath. |
| $\mathrm{G}_{1}+\mathrm{G}_{2}$. | 30 | +.000,0189,8 | -0.637 | -0.571 | $-7 \times 1$ |
| $\mathrm{H}_{1}-\left(\mathrm{G}_{1}+\mathrm{K}_{1}\right)$ | 36 | -.000,0212,5 | +0.068 | +0.407 | -7.11 |
| $\mathrm{H}_{1}+\mathrm{H}_{2}$. | 35 | + 000,0027,4 | -0.435 | +0.065 | -6.72 |
| $\mathrm{I}-\left(\mathrm{H}_{1}+\mathrm{K}_{8}\right)$ | 40 | -.000,0185,9 | +0.151 | -0.013 | +2.96 |
| $\mathrm{J}_{1}-\left(\mathrm{I}^{1}+\mathrm{K}_{3}+\mathrm{J}_{2}\right)$ | 49 | - 000,0062,2 | +0.139 | -0.072 | +3.05 |
| $\mathrm{J}_{1}+\mathrm{L}$. . . . | 48 | + 000,0257,1 | +0.060 | -0.256 | -3.91 |

From the five preceding tables we obtain the following table of the average error generated in ten consecutive triangles in each of the five great figures, the three sections which were mainly executed with the best instruments being shown first, and then the two others, into which work executed with the smaller and less accurate instruments largely enters.

| Section. | Average error generated in ten triangles. |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Log. side-length. | Latitude. | Longitude. | Azimuth. |
| North-West Quadrilateral <br> South-East <br> Southern Trigon | $\begin{array}{r}  \pm \cdot 000,0008,7 \\ \cdot 000,0004,7 \\ \cdot 000,0006,1 \end{array}$ | $\pm 0.015$ 0.011 0.009 | $\begin{array}{r}  \pm 0.016 \\ 0.018 \\ 0.026 \end{array}$ | $\prime \prime$ $\pm 0^{\prime \prime} 22$ 0.25 0.48 |
| Average for first class | $\cdot 000,0006,5$ | $0 \cdot 012$ | $0 \cdot 020$ | $0 \cdot 32$ |
| North-East Quadrilateral. South-West " | $\begin{array}{r} \cdot 000,0031,7 \\ \cdot 000,0039,3 \end{array}$ | $\begin{aligned} & 0.059 \\ & 0.063 \end{aligned}$ | $\begin{aligned} & 0.038 \\ & 0.058 \end{aligned}$ | $\begin{aligned} & 0.83 \\ & 1.33 \end{aligned}$ |
| Average for second class . . | $\cdot 000,0035,5$ | $0 \cdot 061$ | 0.048 | 1.08 |

The magnitudes of the angular corrections range from 0 to $4^{\prime \prime}$; the following table shows the number of corrections of each magnitude which were applied in each section.

| Section. | Total number of angles. | Corrections, and number of angles receiving them. |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{gathered} 0 \\ \text { to } \\ \cdot 1^{\prime \prime} \end{gathered}$ | $\begin{aligned} & \cdot 1^{\prime \prime} \\ & \text { to } \\ & 2^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \cdot 2^{\prime \prime} \\ & \text { to } \\ & \cdot 3^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 3 \prime \\ & \text { to } \\ & 4^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 4^{\prime \prime} \\ & \text { to } \\ & 5^{\prime \prime} \end{aligned}$ | $\begin{gathered} \cdot 5^{\prime \prime} \\ \text { to } \\ 1^{\prime \prime} \cdot 0 . \end{gathered}$ | $\begin{gathered} 1^{\prime \prime} \cdot 0 \\ \text { to } \\ 1^{\prime \prime} \cdot 5 . \end{gathered}$ | $\begin{gathered} 1^{\prime \prime \cdot} \cdot 5 \\ \text { to } \\ 2^{\prime \prime} \cdot 0 . \end{gathered}$ | $\begin{gathered} 2^{\prime \prime} \cdot 0 \\ \text { to } \\ 3^{\prime \prime} \cdot 0 . \end{gathered}$ | $\begin{gathered} 3^{\prime \prime} \cdot 0 \\ \text { to } \\ 4^{\prime \prime} \cdot 0, \end{gathered}$ |
| North-Western Quadrilateral | 1650 | 1511 | 116 | 20 | 2 | 1 |  |  |  |  |  |
| South-Eastern ", | 831 | 706 | 109 | 13 | 2 | 1 |  |  |  |  |  |
| Southern Trigon . . . . . | 909 | 777 | 103 | 12 | 4 | 2 | 11 | 2 |  |  |  |
| North-Eastern Quadrilateral . | 1719 | 780 | 390 | 2.29 | 106 | 58 | 128 | 17 | 10 |  |  |
| South-Western " | 516 | 103 | 115 | 109 | 78 | 31 | 49 | 11 | 9 | 9 | 2 |

It will be seen that the relative magnitude of the closing errors, and of the corrections to the angles, fully justify the decision which was made regarding the order in which the several sections of the triangulation should be taken up for final reduction.

The preceding details sufficiently indicate the general character of the triangulation ; but some further information regarding it is still necessary.

The geodetic calculations were made on the assumption that the Earth's figure is spheroidal and of the following dimensions :-

> Major axis $20,922,932$ feet
> Minor axis $20,853,375$ "
> Ellipticity $\frac{1}{800 \cdot 80}$.

These values were originally computed by Colonel Everest from such data as were available in 1830. He computed a second set of constants after the completion of the Great Arc in 1844, but they have never been employed in this Survey, and are now known to be less accurate than the first set. They are given in the following table, with the constants determined by other investigators :-

|  | Major axis in feet <br> $a$. | Minor axis in feet b. | $a-b$. | $\frac{a-b}{a}$ |
| :---: | :---: | :---: | :---: | :---: |
| Everest's second set . . . | 20,920,902 | 20,853,642 | 67,260 | $\frac{1}{311 \cdot 04}$ |
| Laplace | 20,919,768 | 20,852,822 | 66,946 | $\frac{1}{318.80}$ |
| AIry | 20,923,713 | 20,853,810 | 69,903 | $\frac{1}{200 \cdot 38}$ |
| Bessel . | 20,923,600 | 20,853,656 | 69,944 | $\frac{1}{509 \cdot 15}$ |
| Clarke's first sei | - $20,926,348$ | 20,855,233 | 71,115 | 304\%8 |
| Clarire's last set | 20,926,202 | 20,854,895 | 71,307 | $\frac{1}{308.47}$ |

The unit of length of the Survey is the 10 -foot standard iron bar A whose relations to other well-known standards are as follows at the temperature of $62^{\circ} \mathrm{F}$.

Indian 10-foot bar $\mathbf{A}=3.33331886$ in English standard yards.

$$
\begin{array}{ll}
" & =1351 \cdot 1+821 \text { in lines of the toise. } \\
" & "
\end{array}
$$

The adopted origin of the Survey is the trigonometrical station of Kalianpur, near the Sironj base-line, in Central India. The latitude of this station was determined by Everest as $24^{\circ} 7^{\prime} 11^{\prime \prime} \cdot 26$ from astronomical observations taken in 1825 and 1839-41, and that value is the basis of all the geodetic latitudes of the Survey. Subsequently additional observations were taken at Kalianpur, and the whole were finally reduced with the latest and best values of star places, as described in vol. 11, ' Astronomical Latitudes;' the value $24^{\circ} 7^{\prime} 11^{\prime \prime} \cdot 10$ was then finally obtained. Thus a correction of $-0^{\prime \prime} \cdot 16$ has to be applied to all the geodetic latitudes to reduce them to the final value of the latitude of the origin.

The azimuth was determined astronomically at Kalianpur, and eventually at a large number of stations around; a comparison of the subsequent determinations with the initial azimuth, through the triangulation, led to its receiving a correction of $-1^{\prime \prime} \cdot 1$ for local attraction ; see section 4, chap. xi., vol. 2. Thus corrected it became the fundamental azimuth of the final geodetic calculations.

As the elements of the Earth's figure adopted for the geodetic calculations so long ago as 1830 are not so accurate as those lately determined from fuller data, it is necessary here to give as much of the formulse employed as is required to facilitate the conversion of the given latitudes and longitudes to the values which would have been obtained had other elements been employed. The formulæ were devised by Puissant for the solution of the problem,-Given the latitude and longitude of a station $A$, the azimuth at $A$ of a station $B$, and the distance of $B$ from $A$, to find the latitude and longitude of B , and the azimuth of A at B .

Let $a$ and $b$ be the major and the minor semi-axes, $e$ the eccentricity $=\sqrt{\left(a^{2}-b^{2}\right) / \alpha^{2}}$, $\rho$ the radius of curvature to the meridian in latitude $\lambda, \nu$ the normal to the meridian in the same latitude, $\lambda$ and $L$ the given latitude and longitude of A , and $A$ the given azimuth of B at $\mathrm{A}, \lambda+\Delta \lambda$ and $L+\Delta L$ the required latitude and longitude of B , and $B$ the required azimuth of A at B ; also let $c$ be the distance between A and B at the sea level. Then, all azimuths being measured from the south, we have

$$
\begin{gathered}
\Delta \lambda^{\prime \prime}=\left\{\begin{array}{l}
-\frac{c}{\rho} \cos A \operatorname{cosec} 1^{\prime \prime} \\
-\frac{1}{2} \frac{c^{2}}{\rho \cdot \nu} \sin ^{2} A \tan \lambda \operatorname{cosec} 1^{\prime \prime} \\
-\frac{3}{4} \frac{c^{2}}{\rho \cdot \nu} \frac{e^{2}}{1-e^{2}} \cos ^{2} A \sin 2 \lambda \operatorname{cosec} 1^{\prime \prime} \\
+\frac{1}{6} \frac{c^{3}}{\rho \cdot \nu^{2}} \sin ^{2} A \cos A\left(1+3 \tan ^{2} \lambda\right) \operatorname{cosec} 1^{\prime \prime}
\end{array}\right. \\
\Delta L^{\prime \prime}=\left\{\begin{array}{l}
-\frac{c \sin A}{\nu \cos \lambda} \operatorname{cosec} 1^{\prime \prime} \\
+\frac{1}{2} \cdot \frac{c^{2}}{\nu^{2}} \frac{\sin 2 A \tan \lambda \operatorname{cosec} 1^{\prime \prime}}{\cos \lambda} \\
-\frac{1}{6} \frac{c^{3}}{\nu^{3}} \frac{(1+3 \tan \lambda) \sin 2 A \cos A}{\cos \lambda} \operatorname{cosec} 1^{\prime \prime} \\
+\frac{1}{3} \frac{c^{3} \sin ^{3} A \tan \lambda}{\nu^{3} \lambda} \operatorname{cosec} 1^{\prime \prime}
\end{array}\right. \\
B-(\lambda+A)=\left\{\begin{array}{l}
-\frac{c}{\nu} \sin ^{2} A \tan \lambda \operatorname{cosec} 1^{\prime \prime} \\
+\frac{1}{4} \frac{c^{2}}{\nu^{2}}\left\{1+2 \tan 2 \lambda+\frac{e^{2} \cos 2}{1-e^{2}}\right\} \sin 2 A \operatorname{cosec} 1^{\prime \prime} \\
-\frac{c^{3}}{\nu^{3}}\left(\frac{5}{6}+\tan \lambda\right) \frac{\tan \lambda}{2} \sin 2 A \cos A \operatorname{cosec} 1^{\prime \prime} \\
+\frac{1}{6} \frac{c^{3}}{\nu^{3}} \sin ^{3} A \tan \lambda\left(1+2 \tan ^{2} \lambda\right) \operatorname{cosec} 1^{\prime \prime} .
\end{array}\right.
\end{gathered}
$$

Each quantity is the sum of four terms which are respectively symbolised with the aid of the four prefixes $\delta_{1}, \delta_{2}, \delta_{3}$, and $\delta_{4}$; thus we have

$$
\begin{aligned}
\Delta \lambda & =\delta_{1} \lambda+\delta_{2} \lambda+\delta_{3} \lambda+\delta_{4} \lambda \\
\Delta L & =\delta_{1} L+\delta_{2} L+\delta_{3} L+\delta_{4} L \\
\Delta A & =\delta_{1} A+\delta_{2} A+\delta_{3} A+\delta_{4} A
\end{aligned}
$$

Differentiating with respect to $\rho, \nu$ and $e$ we have

$$
\begin{aligned}
& d . \Delta \lambda=-\Delta \lambda \frac{d \rho}{\rho}-\delta_{2} \lambda \frac{d \nu}{\nu}-\delta_{3} \lambda\left(\frac{d \nu}{\nu}-\frac{2 d e}{\left(1-e^{2}\right) e}\right)-2 \delta_{4} \lambda \frac{d \nu}{\nu} . \\
& d . \Delta L=-\Delta L \frac{d \nu}{\nu}-\delta_{2} L \frac{d \nu}{\nu}-\left(\delta_{3} L+\delta_{4} L\right) 2 \frac{d \nu}{\nu}
\end{aligned}
$$

Also

$$
\begin{aligned}
\frac{d \rho}{\rho} & =-\frac{1}{a}\left\{d a-2 d b-3(d a-d b) \sin ^{2} \lambda\right\} \\
\frac{d \nu}{\nu} & =+\frac{1}{a}\left\{d a+(d a-d b) \sin ^{2} \lambda\right\} \\
\frac{d e}{e} & =\frac{1}{d e^{2}}\{d a-d b\}
\end{aligned}
$$

The most probable values of the Earth's axes at present known are those in Colonel Clarke's ' Geodesy' from his latest investigations. According to them, the errors of the elements of the Indian Survey are $d a=-3270$ feet, and $d b=-1520$ feet. Inserting these values in the above expressions we have, with sufficient approximation,

$$
\begin{aligned}
d . \Delta \lambda & =-\left\{\cdot 000,011-\cdot 000,251 \sin ^{2} \lambda\right\} \Delta \lambda+\cdot 00795 \sin 2 \lambda, \\
d . \Delta L & =+\left\{\cdot 000,156+\cdot 000,084 \sin ^{2} \lambda\right\} \Delta L .
\end{aligned}
$$

The corresponding corrections to arcs of $1^{\circ}$ in latitude and $1^{\circ}$ in longitude, for successive parallels $5^{\circ}$ apart, are as follow :-

| Latitude <br> correction. | Parallel. | Longitude <br> correction. |
| :---: | :---: | :---: |
| +0.031 | 0 | -0.565 |
| +0.008 | 10 | -0.572 |
| +0.026 | 15 | -0.583 |
| -0.070 | 20 | -0.598 |
| -0.128 | 25 | -0.616 |
| -0.194 | 30 | -0.637 |
| -0.267 | 35 | -0.662 |

Corresponding corrections to the primarily computed latitudes and arcs of longitude are given hereafter in the investigations of local attraction.

## The Meridional Arcs.

Lambton's first meridional arc was a short one, only $1^{\circ} \cdot 35^{\prime}$ in length, which was situated about 35 miles to the west of Madras. His subsequent arcs were all portions of the Great Arc on the central meridian of India. His operations were at first embarrassed by the result that the lengths of the meridional degrees appeared to decrease instead of to increase with the latitude, but he very soon found that this was due to deflections of the plumb-line at the astronomical stations which were caused by local attractions; this anomaly disappeared when the length of the arc was increased and new latitude stations were introduced

He employed a zenith sector, by Ramsden, which was considered a beautiful instrument for that time. It consisted of an arc of $20^{\circ}$, graduated to a radius of 60 inches, and a telescope of a focal length of 62 inches; the arc was fixed at the bottom of the frame of the instrument, while the telescope was suspended at its object end on the summit of the frame, from a horizontal axis which was placed exactly over the centre of the graduated arc. Thus, in pointing to zenith stars the eye end of the telescope moved over the face of the arc, and it was furnished with a microscope for reading the graduations. The instrument could be rotated horizontally, and stars were observed with the telescope in the two positions east and west on alternate nights. The adjustment for verticality was effected with a plummet suspended over the outer end of the horizontal axis carrying the telescope ; an arrangement was provided for setting the thread of the plummet exactly opposite the dot which indicated the centre of the axis; the thread coincided with the central line of the graduated arc when the instrument was truly vertical. A full description and a drawing of the instrument are given in vol. 11.

Now, in all graduated instruments, the errors of graduation have usually most importance. There are no means of controlling such errors for partial arcs, but, for complete circles, they may be very effectively controlled by employing a number of equidistant microscopes to read the circle, and shifting their positions systematically so as eventually to obtain circle readings at a large number of equidistant points, according to the method already described as having been introduced by Everest for the measurements of the horizontal angles of the survey. Thus, in conformity with his treatment of theodolites, Everest abandoned the zenith sector and introduced two astronomical circles in its place. They were constructed by Messrs. Trougrion and Sims, in 1835, in the form of alt-azimuth instruments, with large vertical and small horizontal circles. Each instrument was fitted with a pair of graduated vertical circles, 36 inches in diameter, which were bolted together with the telescope between them; the supporting pillars carried a pair of fixed microscopes, $180^{\circ}$ apart on one side, and, on the other side, a pair of flying microscopes, also $180^{\circ}$ apart, which could be set on various points of the circle through a range of $70^{\circ}$; change of face was made systematically by turning the instrument $180^{\circ}$ in altitude and azimuth, and
systematic reversals of the pivots in the $Y$ 's were also made in order that the circles might be brought in turn opposite the fixed and the flying microscopes. For full details and drawings, see vol. 11.

With these two astronomical circles, Everest and Wavgh determined the arcs of amplitude of the two northern sections of the Great Arc, in the years 1839-41; simultaneous observations were taken to eighteen pairs of stars, situated half to the north and half to the south of the zeniths of the two stations, and the same stars were observed at both stations in order to eliminate errors of star's place.

Astronomical observations were then suspended for more than 20 years, during which the strength of the department was wholly devoted to the triangulation. In 1863 two astronomical parties were formed for the purpose of making the recent triangulation subservient to geodesy by the measurement not only of additional meridional arcs, but of longitudinal arcs also, which the recent completion of electric telegraph lines all over the country had rendered possible.

The two astronomical circles being very heavy and difficult to carry about, two new zenith sectors were designed by Colonel Strange-a retired officer of this Surveyand were constructed under his superintendence for observing latitudes. Full descriptions and drawings of them are given in vol. 11, but they must be very briefly described here. Each instrument has a pair of sectors, of amplitude $55^{\circ}$ with a radius of 18 inches, lying opposite each other in a single casting of brass; they were graduated by Troughton and Simm's celebrated dividing engine, on strips of silver hammered into grooves in the brass. The sector plate is permanently attached to the telescope, at right angles, so that the two move together. The sectors are read by four microscopes which are attached to a fixed horizontal frame, in pairs, $20^{\circ}$ apart, one pair for each sector. The extreme range of star observation is $17 \frac{1}{2}^{\circ}$ on either side of the zenith. Every star is observed in the two positions, "telescope east" and "telescope west" in rapid succession, the first observation being taken about 20 seconds before, and the last as much after, transit over the meridian.

One or other of the two astronomical circles and the two new zenith sectors was employed in all the latitude observations up to the field season of 1890-91. A zenith telescope was then introduced for the first time ; it has no vertical arcs, but determines the latitudes by measuring the differences of the zenith distances of north and south stars of nearly equal zenith distance with a micrometer in the eye-piece, in accordance with what is known as the Talcott method of observation.

The latitude observations after the year 1862 were first taken at additional stations on the Great Arc at distances of about $1^{\circ}$ apart; afterwards at groups of stations composed of one principal and a few secondary stations situated around and within a few miles of the principal, to ascertain whether there were any local variations of density under the surface of the ground to produce deflections of the plumb-line as had been met with on the plains of Moscow. Additional stations were introduced on and in the neighbourhood of the Himalayan Mountains to ascertain the influence of
their attraction. Thus forty-eight latitude stations were added to the primary eleven on the Great Arc. Latitude observations have also been made on the following chains of triangles-the Rahun and the Gurhagarh series and their southern extension through the Khanpisura series to Mangalore, which form an important meridional combination to the west of the Great Arc ; the Amua, Jabalpur, and the Madras meridional series, forming a similar combination to the east; the Jodhpore series and its extension southwards to Bombay; the Calcutta-Kurrachee longitudinal series, and the Bombay and the Bider longitudinal series.

Latitudes have been observed at 161 stations in all, for a considerable majority of which full details are given in vol. 11; for the remainder the details have not yet been published, but I am able to give a synopsis of the results which has been communicated to me in manuscript.

For the computation of the latitudes in vol. 11 a very careful investigation of the places of all the stars employed was made from twenty-one star catalogues, of which the five Greenwich Catalogues for 1840, 1845, 1860, 1864, and 1872, and Mädler's for 1850 were mainly depended upon. Whenever a star was found in these six catalogues its place was taken from those which gave results within $1^{\prime \prime}$ of the mean ; if at least three gave accordant results the mean was accepted; if the star only appeared in two of them, then other catalogues were employed. A catalogue was prepared for the computation which shows the value given by each authority, as well as that which was finally adopted.

There are 111 latitude stations in vol. 11; at 94 of these the observations were taken with one or other of the two astronomical circles by Troughton and Simms, or the two zenith sectors by Strange; at five they were taken with one of these instruments and also with Ramsden's zenith sector, at five more with Ramsden's zenith sector only; in five cases they were taken with the 18 -inch vertical circle of one of the 36 -inch or the 24 -inch theodolites; and in two cases, in the Himalayan Mountains, with the 10 -inch vertical sectors of a 14 -inch theodolite.

As it is a well known fact that some instruments have a tendency to exhibit more or less constant differences between latitudes deduced from north stars and those deduced from south stars, it has been generally a rule to observe as many north as south stars at each station. In the final reductions, even when this rule has not been followed, a value of the latitude is always deduced from the north stars and annther from the south stars, and the arithmetical mean of the two is adopted for the final value.

Astronomical Circle No. 2, and Strange's Zenith Sector No. 2, have given results throughout which do not show constant differences between north and south stars. The two sister instruments have, however, shown marked constant differences, which must be here noticed, though it is believed that the errors of each are cancelled in the means.

Astronomical Circle No. 1 gives the following values of North-South :-


The instrument must have met with some mishap between February and October, 1866, but the nature of the mishap has baffled all search. As a check the latitude was re-observed at two of the stations of the last group with Astronomical Circle No. 2, and results were obtained very closely coinciding with the means of north and south stars originally deduced, showing that the latitudes obtained with the instrument are quite reliable. It was, however, set aside, and has not been used since 1867.

Strange's Zenith Sector No. 1 has invariably given greater latitudes by south than by north stars ; the difference increases systematically with the zenith distance, rising from $1^{\prime \prime} \cdot 9$ for a mean zenith distance of $4^{\circ} 16^{\prime}$ to $5^{\prime \prime} \cdot 4$ for $15^{\circ}$. The latitudes by south stars being greatest, the zenith distances are too great; this suggests that the sector may have contracted after having been graduated; the graduation is known to have been performed when the sector was a new casting, which may have undergone some change before finally settling down. The sister sector is believed not to have been graduated until some time after the casting. Whatever the cause, it has been recently shown that the mean of north and south stars is free from error, for the instrument has been employed at three stations both as a zenith sector and as a zenith telescope, and the latitudes obtained from the sectors differed by less than $0^{\prime \prime} \cdot 1$ from the values with the telescope alone.

The following tables give the results of the latitude observations at each station, together with the longitude of the station and its height above the sea-level, the instrument employed, and the year of observation. The first table has been wholly abstracted from vol. 11 ; the second gives results which have not yet been published. A majority of the stations are principal stations of the great triangulation; but in certain localities the latitude has been observed at several points around a principal station whose positions were fixed by secondary triangulation; such points are practically as valuable for geodetic purposes as the principal stations, but they are entered as secondaries in the tables.

Determinations of Latitude, published in Volume 11.



Determinations of Latitude, published in Volume 11-(continued).


Determinations of Latitude, published in Volume 11-(continued).



Determinations of Latitude, published in Volume 11-(continued).


Determinations of Latitude, hitherto unpublished.

| Longitade of station. | Height above sea. | Instrument employed. | Year. | Name of station. | Astronomical latitude. | Pairs of stars. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 77 | 2274 | Zenith Telescope | 1892-93 | Achola . | $181{ }^{\circ} 1444.91 \pm " 08$ | 39 |
| 78 3 | 550 | Z. S. No. 1 | 1893-94 | Agra . | $\begin{array}{lllll} & 27 & 9 & 34.52 & \cdot 06\end{array}$ | 4.3 |
| 7455 | 770 |  |  | Amritsar | $\begin{array}{llll}31 & 38 & 2 \cdot 53 & 08\end{array}$ | 19 |
| 7939 | 1463 | Z. S. No. 2 | 1888-89 | Ankora. | $\begin{array}{ll}192426.68 ~ & 05\end{array}$ | 29 |
| 7949 | 1490 |  |  | Bhimsain | $\begin{array}{ll}20 & 57 \quad 28 \cdot 60\end{array}$ | 34 |
| 7242 | 774 | Z. S. No. 1 | 1893-94 | Bithnok | $275325 \cdot 04 \quad 05$ | 45 |
| 7834 | 1600 | Zenith Telescope |  | Bolarum | $\begin{array}{llll}17 & 30 & 7 \cdot 50 & 06\end{array}$ | 47 |
| 7950 | 1370 | Z. S. No. 2 | 1888-89 | Bolikonda | $\begin{array}{ll}17 & 42 \\ 29 \cdot 12 & 04\end{array}$ | 35 |
| 7251 | 75 | Zenith Telescope | 1892-93 | Bombay (Colaba Observatory) | $\begin{array}{ll}18 & 53 \\ 39 \cdot 16 \quad 06\end{array}$ | 39 |
| 7944 | 983 | Z. S. No. ${ }^{2}$ | 1888-89 | Burgpaili . . . . | $\begin{array}{llll}18 & 54 & 351 & 05\end{array}$ | 31 |
| 7238 | 1065 | Z. S. No. 1 | 1892-93 | Chamu. | $263953.49 \quad 05$ | 24 |
| 7235 | $9: 53$ | ", |  | Chaniána | $\begin{array}{lllll}24 & 6 & 25 \cdot 45 & 04\end{array}$ | 25 |
| 80 | 300 |  | 1890-91 | Dánapa. | $1555 \quad 59 \cdot 70$-05 | 45 |
| 7957 | 195 | Zenith Telescope |  | Darutippa. | $\begin{array}{lllll}15 & 0 & 33 \cdot 52 & 04\end{array}$ | 61 |
| 781 | 1958 | " | 1892-93 | Dehra Dun Base Line, East end | $301637 \cdot 29$-08 | 19 |
| 7059 | 490 | Z. S. No. 1 | 1893-94 | Dehra Din Panah | $\begin{array}{llll}30 & 33 & 59.62 & 09\end{array}$ | 25 |
| 7214 | 443 |  | 1892-93 | Deera | $2415 \quad 21 \cdot 18 \quad 05$ | 24 |
| 7412 | 2939 | Zenith Telescope |  | Dhauleshvar | $\begin{array}{llll}18 & 25 & 42.91 & 05\end{array}$ | 57 |
| 808 | 245 | Z. S. No. 2 | 1888-89 | Dhúlipalla. | $\begin{array}{lllll}16 & 25 & 53.48 & 06\end{array}$ | 28 |
| 7935 | 967 |  |  | Díwai . | $\begin{array}{llll}19 & 49 & 26.90 & 05\end{array}$ | 30 |
| 80 72 48 | 292 5200 | Zenith Telescope | 1890-91 | Gudali . | 14 1 $10 \cdot 65$ .06 <br> 24 37   <br> 1780    | 45 |
| 7234 | 772 | Z. S. No. 1 | 1892-93 | Guru Sikkar | 24 27 27 16 $6 \begin{array}{lll}37 & 80 & \cdot 04 \\ .04\end{array}$ | $\stackrel{21}{25}$ |
| 7546 | 2610 | Zenith Telescope | " | Kanheri | $\begin{array}{llll}18 & 29 & 21.94 & 05\end{array}$ | 61 |
| 7449 | 2751 |  |  | Khanpisura* | $\begin{array}{llll}18 & 45 & 22 \cdot 57 & 05\end{array}$ | 57 |
| 7242 | 603 | Z. S. No. 1 | 1893-94 | Khirsar . | $\begin{array}{llll}28 & 29 & 43.77 & 06\end{array}$ | 48 |
| 7948 | 458 | Zenith Telescope | 1890-91 | Kistáma | $\begin{array}{lllll}14 & 27 & 12 \cdot 30 & 05\end{array}$ | 45 |
| 72.2 | 470 | Z. S. No. 1 | 1893-94 | Ladimsir | $\begin{array}{llllll}29 & 21 & 39 \cdot 84 & 06\end{array}$ | 62 |
| 8011 | 1400 | Z. S. No. 2 | 1886-87 | Lingmára | $\begin{array}{lllll}21 & 42 & 55 \cdot 41 & 05\end{array}$ | 33 |
| 7335 | 4121 | Zenith Telescope | 1892-93 | Mandvi. | $\begin{array}{llllll}18 & 37 & 48.09 & 05\end{array}$ | 53 |
| 8017 | 54 | " | 1890-91 | Madras Observatory | $\begin{array}{lllll}13 & 4 & 9 \cdot 88 & \cdot 07\end{array}$ | 39 |
| 8014 | 250 | " | " | Madras, St. Thomas' Mount | $\begin{array}{llll}13 & 0 & 20 \cdot 66 & .08\end{array}$ | ¿9 |
| 7129 | 420 | Z. S. No. 1 | 1893-94 | Mooltan | $\begin{array}{llll}30 & 10 & 56.23 & 07\end{array}$ | 47 |
| 7946 | 1144 | Z. S. No. 2 | 1888-89 | Niálamuri | $\begin{array}{lllll}17 & 1 & 25.96 & \cdot 0.4\end{array}$ | 35 |
| 7619 | 2289 | Zenith Telescope | 1892-93 | Nitali | $\begin{array}{lllll}18 & 17 & 2 \cdot 80 & 05\end{array}$ | 61 |
| 805 | 250 | Zonith Tolescopo | 1890-91 | Ongole | $\begin{array}{lllll}15 & 29 & 52 \cdot 90 & 05\end{array}$ | 47 |
| 8115 | 684 | " | 1893-94 | Parampúdi | $\begin{array}{llll}17 & 12 & 32 \cdot 62 & 06\end{array}$ | 43 |
| 7838 | 2090 |  |  | Pirmulo | $\begin{array}{llll}17 & 52 & 58.48 & 07\end{array}$ | 48 |
| 787 | 2900 |  | 1892-93 | Rajpúr . | $\begin{array}{lllll}30 & 23 & 9.33 & \cdot 11\end{array}$ | 10 |
| 7947 | 1070 | Z. S. No. 2 | 1886-87 | Rajuli . | ${ }^{20} 121251 \cdot 29$-04 | 40 |
| 7934 | 1779 |  | 1888-89 | Rámgír. | 18 35 26.96 04 | 30 |
| 7287 | 846 | Z. S. No. 1 | 1892-93 | Samdári | $\begin{array}{lllll}25 & 48 & 59.61 & 04\end{array}$ | 27 |
| 8244 | 2142 | Zenith Telescope | 1893-94 | Sanjib - | $\begin{array}{lllll}17 & 31 & 12.32 & \cdot 07 \\ 22 & 12 & 50.71 & \cdot 05\end{array}$ | 41 |
| 805 | 1100 | Z. S. No. 2 | 1886-87 | Sarandi Pat | $\begin{array}{llllll}22 & 12 & 50.71 & 05\end{array}$ | 25 |
| 8059 | 714 | Zenith Telescope | 1893-94 | Singáwáram | $\begin{array}{lllll}17 & 45 & 8 \cdot 69 & \cdot 07\end{array}$ | 44 |
| 8022 | 1237 | Z. S. No. 2 | 1886-87 | Sítápár | 21 24 43387 05 | 32 |
| 7248 | 250 | Z. S. No. 1 | 1892-93 | Sonáda . |  | 30 |
| 7217 | 470 | ,. | 1893-94 | Tela . |  | 44 |
| 72 725 | 856 |  | 1892-93 | Thob | $\begin{array}{lllll}26 & 3 & 2 \cdot 94 & \cdot 05 \\ 17 & 3 & \end{array}$ | 24 |
| 7925 | 1664 | Zeuith Telescope | 1893-94 | Vánákonda | $\begin{array}{lllll}17 & 36 & 0.30 & \cdot 06\end{array}$ | 46 |
| 8322 | 500 |  | " | Waltair | 1743 $20 \cdot 38$ <br> 05  | 52 |

*The latitude of Khanpisura obtained in 1870 being $18^{\circ} 45^{\prime} 22^{\prime \prime} \cdot 63$ (vol. 9 ), the value to be adopted is $18^{\circ} 45^{\prime} 22^{\prime \prime} 60$.

The results given in the two last tables have now to be arranged for the meridional and the longitudinal arcs to which they respectively appertain; also the geodetic latitudes must be set forth by the side of the astronomical, that the differences between the two sets of results may be exhibited.

Occasional combinations of results must be made in order to place the data in the most convenient form for future investigations ; thus for all localities in which groups of stations have been observed, with a view to cancelling the effects of local attraction as much as possible, it is obviously necessary to employ the mean of all the astronomical and the mean of all the gendetic values, instead of the individual values; the hypothetical points thus introduced are as serviceable for geodetic investigations as the actual points of observation. And this process must be extended to wider regions, where there was no intentional grouping of stations; thus where several astronomical latitudes are forthcoming within a meridional belt of moderate breadth-say not much exceeding $1^{\circ}$, so as not to be sensibly influenced by the normal flattening between the equator and the poles-the mean of these will obviously be less affected by local attraction than the single values, and will therefore represent a truer value of the astronomical latitude; either this must be done or some of the astronomical data must be arbitrarily rejected. For example: on the Great Arc there are five stations between the parallels of $13^{\circ}$ and $14^{\circ} 6^{\prime}$; three of them are Lambron's stations, viz., Doddagunta to the south showing northerly attraction, and Bommasundra and Pavagada to the north, showing southerly attraction; Lambton expressly stated his belief that the plumb-line was deflected in opposite directions by intermediate masses between the north and south stations; but in their investigations of the figure of the earth, both Bessel and Clarke have rejected the northern stations and employed the southern one only. Since then two southern stations have been added, at the ends of the Bangalore base-line near Doddagunta, which also show northerly attraction ; and it now seems obviously necessary to accept the mean of all the stations or to reject them all, rather than to accept some and reject the others. Thus the principle of grouping has, whenever practicable, been carried through the following tabular presentation of astronomical and geodetic latitudes.

Separate tables have been prepared for the Great Arc, for the meridians of Madras, Mangalore, and Bombay ; for the Calcutta and the Bombay and Bider longitudinal triangulations, and for the stations influenced by Himalayan attraction. As the latest astronomical value of the latitude of the station of origin, Kalianpur, has been found
to be $0^{\prime \prime} \cdot 16$ less than the value employed for the geodetic calculations, a correction of $-0^{\prime \prime} \cdot 16$ has been applied to all the geodetic latitudes in the published volumes and the addenda to obtain the values given in these tables.

In the tables the geodetic values are those which were obtained with the elements of the Earth's figure, which Everest computed in 1830, and have ever since been employed in all the geodetic calculations of the Indian Survey. The quantities A-G are the differences between them and the astronomical values. The quantities $A-G_{c}$ are those which would have been obtained if Clarke's latest elements had been employed instead; they are shown in the table for Himalayan stations and also in subsequent tables.

The last column of these tables gives rough statements as to whether there is any apparent meridional attraction at any station, and if so, whether its direction is northerly or southerly, and its amount small or great. In most instances this information is supplied from the notes of the observers, otherwise it is estimated from the topographical details in the sheets of the Indian Atlas.

Comparisons of Astronomical with Geodetic Latitudes.
Great Arc Meridional Series.




Great Arc Meridional Series-(continued).


GENERAL J. T. Walker on india's contribution to geodest.

Great Arc Meridional Series-(continued).

| Name of station. | Longitade. | Height in feet above sea. | Latitade. |  |  |  |  |  |  | Eye estimate of direction and magnitude of meridional attraction |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. |  |  | Geodetic. |  |  | A-G. |  |
| Ládi | 77 | 1875 | 23 | 83 | $39 \% 10$ |  | $\dot{8}$ | $43 \% 97$ | -4.87 | Uncertain |
| Kaliánpur | 7742 | 1765 | 24 | 71 | 11.10 | 24 | 7 | 11.10 | .. | None |
| $\underset{\text { Pahárgarh }}{\text { Kesri. }}$. | $\begin{aligned} & 77 \quad 44 \\ & 77 \end{aligned}$ | $\begin{aligned} & 1641 \\ & 1487 \end{aligned}$ | 24 | $\begin{aligned} & 56 \\ & 46 \end{aligned}$ | $\begin{array}{r} 6 \cdot 47 \\ 41.57 \end{array}$ |  |  | $\begin{array}{r} 6.76 \\ 35.65 \end{array}$ | $\begin{aligned} & -0.29 \\ & +5.92 \end{aligned}$ | Uncertain <br> None |
| Group 11. | 7744 | .. | 25 | 212 | 24.02 | 25 | 21 | 21.21 | +2.81 |  |
| Usira. Agra. | 77 78 78 | $\begin{aligned} & 810 \\ & 550 \end{aligned}$ | ${ }_{27}^{26}$ | 57 9 | $\begin{array}{r} 0.50 \\ 34: 52 \end{array}$ |  | $\begin{array}{r} 57 \\ 9 \end{array}$ | $\begin{array}{r} 6.06 \\ 39.77 \end{array}$ | $\begin{aligned} & -5 \cdot 56 \\ & -5.25 \end{aligned}$ | Slightly northern |
| Group 12. | 7752 | .. |  | 31 | 17.51 |  | 3 | 22.92 | -5.41 |  |
| Noh . Datairi | $\begin{array}{ll}77 & 41 \\ 77 & 41\end{array}$ | 710 767 | 27 | $\begin{array}{r}50 \\ 43 \\ \hline\end{array}$ | 58.13 58.67 |  | 50 | $\begin{gathered} 52 \cdot 92 \\ 4: 33 \end{gathered}$ | ${ }_{-5.66}^{+0.21}$ | Slightly southern None |
| Group 13. . . | 7741 | .. | 28 | 172 | $25 \cdot 90$ |  | 17 | 28.63 | $-2 \cdot 73$ |  |
| Kaliána | 7742 | 828 | 293 | 304 | $47 \cdot 98$ |  | 30 | 54.54 | -6.56 | Possibly northern to Himalayas. |

Madras Meridional Triangulation.

| Stations. | Longitude. | Height in feet. | Latitude. |  |  | Eye estimate of direction and magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. | Geodetic. | A-G. |  |
| Tíruvendiparam . . | 79 | 125 | $\begin{array}{lll}11 & 44 & 43^{\prime \prime} \cdot 40\end{array}$ | $\begin{array}{lll}11 & 44 & 37\end{array}$ | $+{ }^{\prime \prime} \times 7$ | No attraction |
| Madras, St. Thomas' Mount Madras Observatory | $\begin{array}{ll} 80 & 14 \\ 80 & 17 \end{array}$ | $\begin{array}{r} 250 \\ 54 \end{array}$ | $\begin{array}{rrr} 13 & 0 & 20 \cdot 66 \\ 13 & 4 & 9 \cdot 88 \end{array}$ | $\begin{array}{rrr} 13 & 0 & 14.63 \\ 13 & 4 & 4.00 \end{array}$ | $\begin{aligned} & +6.03 \\ & +5.88 \end{aligned}$ | No attraction |
| Group 1 | $80 \quad 16$ | . | $\begin{array}{lll}13 & 2 & 15.27\end{array}$ | $13 \quad 2 \quad 9331$ | +5.96 |  |
| Gudali <br> Kistama. | $\begin{array}{rr}80 & 4 \\ 79 & 48\end{array}$ | $\begin{aligned} & 292 \\ & 458 \end{aligned}$ | $\begin{array}{rrr}14 & 1 & 10 \cdot 65 \\ 14 & 27 & 12 \cdot 30\end{array}$ | $\begin{array}{rrr}14 & 1 & 9 \cdot 29 \\ 14 & 27 & 14 \cdot 40\end{array}$ | $\begin{array}{r} +1 \cdot 36 \\ -2 \cdot 10 \end{array}$ | Slight southerly <br> None |
| Group 2 | $79 \quad 56$ | - | $\begin{array}{lll}14 & 14 & 11.48\end{array}$ | $\begin{array}{lll}14 & 14 & 11.85\end{array}$ | -0.37 |  |
| Darutippa <br> Ongole | $\begin{array}{rrr}79 & 57 \\ 80 & 5\end{array}$ | 195 250 | $\begin{array}{rrr} 15 & 0 & 33 \cdot 52 \\ 15 & 29 & 52 \cdot 90 \end{array}$ | $\begin{array}{rrr} 15 & 0 & 36 \cdot 31 \\ 15 & 29 & 56 \cdot 69 \end{array}$ | $\begin{array}{r} -2.79 \\ -3.79 \end{array}$ | None <br> Sonthern |
| Group 3 | 801 | - | $\begin{array}{lll}15 & 15 & 13.21\end{array}$ | $\begin{array}{lll}15 & 15 & 16.50\end{array}$ | $-3.29$ |  |
| Dánapa . Dhúlipala | $\begin{array}{ll}80 & 0 \\ 80 & 8\end{array}$ | $\begin{aligned} & 300 \mathrm{P} \\ & 245 \end{aligned}$ | $\begin{array}{lll} 15 & 55 & 59 \cdot 70 \\ 16 & 25 & 53 \cdot 48 \end{array}$ | $\begin{array}{lll} 15 & 55 & 59 \cdot 98 \\ 16 & 25 & 56 \cdot 59 \end{array}$ | $\begin{aligned} & -0.28 \\ & -3.11 \end{aligned}$ | None <br> A little northern |
| Group 4 | $80 \quad 4$ | -• | $\begin{array}{lll}16 & 10 & 56.59\end{array}$ | $\begin{array}{lll}16 & 10 & 58.29\end{array}$ | $-1.70$ |  |

general J. t. walker on india's contribution to geodesy.

Madras Meridional Triangulation-(continued).

| Stations. | Longitade. | Height in feet. | Latitade. |  |  |  | Eye estimate of direction and magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. | Geodetic. |  | A-G. |  |
| Niálamari Bolikonda | $\begin{array}{ll} 79 & 46 \\ 79 & 46 \\ \hline 90 \end{array}$ | $\begin{aligned} & 1144 \\ & 1370 \end{aligned}$ | $\begin{array}{rrr} 17 & 1 & 25 ‘ 96 \\ 17 & 42 & 29 \cdot 12 \end{array}$ | $\begin{array}{lr} 17 & 1 \\ 17 & 42 \end{array}$ | $\begin{aligned} & 33: 47 \\ & 35 \cdot 66 \end{aligned}$ | $\begin{array}{r} -7^{\prime \prime} \cdot 51 \\ -6 \cdot 54 \end{array}$ | Slight northern Insignificant |
| Group 5 | $79 \quad 48$ | $\cdots$ | 17 21 57 | $17 \quad 22$ | 4.57 | $-7.03$ |  |
| Ramgir <br> Burgpaili | $\begin{array}{rl} 79 & 34 \\ 79 & 44 \end{array}$ | $\begin{array}{r} 1779 \\ 983 \end{array}$ | $\begin{array}{rrr}18 & 35 & 26.96 \\ 18 & 54 & 3.51\end{array}$ | 18 18 18 | $\begin{array}{r} 25 \cdot 96 \\ 7.04 \end{array}$ | $\begin{array}{r} +1.00 \\ +3.53 \end{array}$ | Some sonthern Slight southern |
| Group 6 | $79 \quad 39$ | -• | $\begin{array}{lll}18 & 44 & 45 \cdot 23\end{array}$ | $18 \quad 44$ | $46 \cdot 50$ | -1.27 |  |
| Ankora <br> Diwai | 79 39 <br> 79 35 | 1463 967 | $\begin{array}{lll}19 & 24 & 26 \cdot 68 \\ 19 & 49 & 26.90\end{array}$ | $\begin{array}{ll}19 & 24 \\ 19 & 49\end{array}$ | $\begin{array}{r} 34: 59 \\ 32 \cdot 41 \end{array}$ | $\begin{aligned} & -7.91 \\ & -5.51 \end{aligned}$ | Slight southern None |
| Group 7 | $79 \quad 37$ | - | 19 36.56.79 | $19 \quad 37$ | 3.50 | -6.71 |  |
| Rajuli <br> Bhimsain | $\begin{array}{ll} 79 & 47 \\ 79 & 49 \end{array}$ | $\begin{aligned} & 1070 \\ & 1490 \end{aligned}$ | $\begin{array}{lll}20 & 12 & 51 \cdot 29 \\ 20 & 57 & 28.60\end{array}$ | $\begin{array}{ll} 20 & 12 \\ 20 & 57 \end{array}$ | $\begin{aligned} & 55 \cdot 29 \\ & 35 \cdot 80 \end{aligned}$ | $\begin{array}{r} -4.00 \\ -7.20 \end{array}$ | None 1" northerly |
| Group 8 | $79 \quad 48$ | - | $\begin{array}{llll}20 & 35 & 9.95\end{array}$ | $20 \quad 35$ | $15 \cdot 55$ | $-5 \cdot 60$ |  |
| Sitapár . <br> Lingmara <br> Sarandi Pat | $\begin{array}{rr} 80 & 22 \\ 80 & 11 \\ 80 & 5 \end{array}$ | $\begin{aligned} & 1237 \\ & 1400 \\ & 1100 ? \end{aligned}$ | $\begin{array}{lll} 21 & 24 & 43 \cdot 87 \\ 21 & 42 & 55 \cdot 41 \\ 22 & 12 & 50 \cdot 71 \end{array}$ | $\begin{array}{ll} 21 & 24 \\ 21 & 43 \\ 22 & 12 \end{array}$ | $\begin{array}{r} 50 \cdot 38 \\ 2 \cdot 91 \\ 55 \cdot 45 \end{array}$ | $\begin{aligned} & -6.51 \\ & -7.50 \\ & -4.74 \end{aligned}$ | Small northerly Large northerly Large northerly |
| Group 9 | $80 \quad 13$ | .. | $21 \quad 46 \quad 50 \cdot 00$ | 2146 | 56.25 | $-6 \cdot 25$ |  |

GENERAL J. T. WALKER ON INDIA'S CONTRIBUTION TO GEODESY. 783

Madras Meridional Triangulation-(continued).

| Stations. | Longitade. | Height in feet. | Latitade. |  |  | Eye estimate of direction and magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. | Geodetic. | A-G. |  |
| Karaundi Lora . | 80  <br> 80 12 | 1625 1923 | $\begin{array}{lll} 20 & 10 & 45: 007 \\ 23 & 29 & 46.30 \end{array}$ | $\begin{array}{lll}2 \times 3 \\ 23 & 10 & 39.86 \\ 23 & 29 & 41.37\end{array}$ | $\begin{aligned} & +5^{\prime \prime} 21 \\ & +4: 93 \end{aligned}$ | Slight sontherly None |
| Gronp 10. | 807 | -• | $\begin{array}{lll}23 & 20 & 1569\end{array}$ | $23 \quad 20 \quad 10 \cdot 62$ | +5.07 |  |
| Karára . <br> Potenda. | $\begin{array}{rr}81 & 18 \\ 81 & 0\end{array}$ | $\begin{array}{r} 1966 \\ 993 \end{array}$ | $\begin{array}{rrr} 24 & 4 & 42 \cdot 20 \\ 24 & 37 & 24 \cdot 71 \end{array}$ | $\begin{array}{rrr} 24 & 4 & 41 \cdot 85 \\ 24 & 37 & 22.88 \end{array}$ | $\begin{aligned} & +0.35 \\ & +1.83 \end{aligned}$ | Northerly <br> If anything slight soatherly |
| Group 11. | 819 | . | $\begin{array}{lll}24 & 21 & 3\end{array}$ | $\begin{array}{lll}24 & 21 & 2 \cdot 37\end{array}$ | +1.09 |  |
| Pavia. <br> Kanakhera . <br> Dewarsan | $\begin{array}{ll} 80 & 47 \\ 80 & 28 \\ 80 & 21 \end{array}$ | $\begin{aligned} & 481 \\ & 416 \\ & 439 \end{aligned}$ | $\begin{array}{lll} 25 & 27 & 21 \cdot 18 \\ 25 & 51 & 25 \cdot 97 \\ 26 & 15 & 58 \cdot 32 \end{array}$ | 25 27 $17 \cdot 23$ <br> 25 51 $20 \cdot 79$ <br> 26 15 52.73 | $\begin{aligned} & +3 \cdot 95 \\ & +5 \cdot 18 \\ & +5 \cdot 59 \end{aligned}$ | Slight sontherly None Slight northerly |
| Group 12. | $80 \quad 32$ | . | $25 \quad 51 \quad 35 \cdot 16$ | $25 \quad 51 \quad 30 \cdot 25$ | +4.91 |  |
| Etora. <br> Nimkar . | $\begin{array}{ll} 80 & 42 \\ 80 & 32 \end{array}$ | $\begin{aligned} & 429 \\ & 486 \end{aligned}$ | $\begin{array}{rrr} 26 & 54 & 22 \cdot 63 \\ 27 & 21 & 8 \cdot 16 \end{array}$ | $\begin{array}{rrr} 26 & 54 & 17 \cdot 69 \\ 27 & 21 & 7.93 \end{array}$ | $\begin{array}{r} +4 \cdot 94 \\ +0 \cdot 23 \end{array}$ | None None |
| Group 13. . . | $80 \quad 37$ | -• | $\begin{array}{lll}27 & 7 & 45 \cdot 40\end{array}$ | $\begin{array}{lll}27 & 7 & 42.81\end{array}$ | +2.59 |  |
| Jardira | $80 \quad 31$ | 536 | $\begin{array}{lll}27 & 59 & 50.22\end{array}$ | $\begin{array}{lll}27 & 59 & 55.78\end{array}$ | $-5 \cdot 56$ | Possibly slight Himalayan attraction |

Mangalore Meridional Triangulation, combining the Rahun and the Gurhagarh Meridional Series.

| Stations. | Longitade. | Height in feet. | Latitude. |  |  |  | Eye estimate of direction and magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. | Geode | tic. | A-G. |  |
| Mangalore . | $7{ }^{7} 4$ | 186 |  | $12 \quad 52$ | 14:60 | $+3^{\prime \prime} 16$ | Slightly northern |
| Koramur <br> Honnavalli . | $\begin{array}{ll}75 & 1 \\ 75 & 13\end{array}$ | 2527 2775 | $\begin{array}{rrr}14 & 8 & 1 \cdot 71 \\ 14 & 16 & 3076\end{array}$ | $\begin{array}{rr}14 & 8 \\ 14 & 16\end{array}$ | $\begin{array}{r} 6 \cdot 43 \\ 32 \cdot 30 \end{array}$ | $-4 \cdot 72$ $-1: 54$ | Northern <br> Southern |
| Group 1 . | $75 \quad 7$ | -• | $14 \quad 12 \quad 16.24$ | $14 \quad 12$ | $19 \cdot 37$ | $-3 \cdot 13$ |  |
| Knndgol. <br> Navalúr . | $\begin{array}{rrr}75 & 17 \\ 75 & 6\end{array}$ | $\begin{array}{r} 2147 \\ 2445 \end{array}$ | $\begin{array}{lll} 15 & 15 & 14 \cdot 46 \\ 15 & 25 & 28 \cdot 48 \end{array}$ | $\begin{array}{ll} 15 & 15 \\ 15 & 25 \end{array}$ | $\begin{aligned} & 15 \cdot 12 \\ & 31 \cdot 01 \end{aligned}$ | $\begin{aligned} & -0.66 \\ & -2.53 \end{aligned}$ | Slightly sonthern Slightly southern |
| Group 2 . | 7511 | . | $\begin{array}{lll}15 & 20 & 21.47\end{array}$ | $15 \quad 20$ | 23.07 | $-1 \cdot 60$ |  |
| Mavinhúnda Majála | $\begin{array}{ll}74 & 50 \\ 74 & 29\end{array}$ | $\begin{aligned} & 2582 \\ & 2613 \end{aligned}$ | $\begin{array}{rrr} 16 & 25 & 4: 47 \\ 16 & 46 & 55 \cdot 45 \end{array}$ | $\begin{array}{ll} 16 & 25 \\ 16 & 46 \end{array}$ | $\begin{array}{r} 4.03 \\ 56.66 \end{array}$ | $\begin{array}{r} +0.44 \\ -1.21 \end{array}$ | None Slightly southern |
| Group 3 | $74 \quad 40$ | - | $\begin{array}{lll}16 & 35 & 59.96\end{array}$ | 1636 | $0 \cdot 35$ | -0.39 |  |
| Kem . Khanpisura | $\begin{array}{ll} 75 & 21 \\ 74 & 49 \end{array}$ | $\begin{aligned} & 1951 \\ & 2751 \end{aligned}$ | $\begin{array}{lll} 18 & 10 & 45 \cdot 68 \\ 18 & 45 & 22 \cdot 60 \end{array}$ | $\begin{array}{ll} 18 & 10 \\ 18 & 45 \end{array}$ | $\begin{aligned} & 48 \cdot 74 \\ & 30 \cdot 49 \end{aligned}$ | $\begin{aligned} & -3.06 \\ & -7.89 \end{aligned}$ | None <br> Slightly northern |
| Groap 4 | $75 \quad 5$ | . | $\begin{array}{lll}18 & 28 & 4\end{array}$ | $18 \quad 28$ |  | -5.48 |  |

Mangalore Meridional Triangulation, combining the Rahun and the Gurhagarh Meridional Series-(continued).

| Stations. | Longitude. | Height in feet. | Latitude. |  |  |  |  | Eye estimate of direction and :magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astrono | mical. | Geode |  | A-G. |  |
| Dhaigaon Valvádi . | $\begin{array}{ll} 75 & 1 ' 5 \\ 75 & 14 \end{array}$ | 1553 1125 | $\begin{array}{ll} 19 & 30 \\ 20 & 44 \end{array}$ | $\begin{aligned} & 30!82 \\ & 21 \cdot 27 \end{aligned}$ | $\begin{array}{ll} \circ & 30 \\ 19 & 30 \\ 20 & 44 \end{array}$ | $\begin{aligned} & 34 " 8 \\ & 27.57 \end{aligned}$ | $\begin{aligned} & \quad A^{\prime} \cdot 06 \\ & -6 \cdot 30 \end{aligned}$ | Slightly northern <br> Very slightly northern |
| Group 5 | $75 \quad 15$ | - | 207 | 26.05 | $20 \quad 7$ | 31.23 | -5.18 |  |
| Thikri <br> Harnása <br> Deo Dongri. | $\begin{array}{ll} 75 & 27 \\ 75 & 36 \\ 75 & 35 \end{array}$ | $\begin{array}{r} 851 \\ 1816 \\ 1727 \end{array}$ | $\begin{array}{rr}22 & 1 \\ 22 & 47 \\ 23 & 26\end{array}$ | $\begin{array}{r} 3 \cdot 92 \\ 26 \cdot 80 \\ 43 \cdot 17 \end{array}$ | $\begin{array}{lr}22 & 1 \\ 22 & 47 \\ 23 & 26\end{array}$ | $\begin{array}{r} 2 \cdot 61 \\ 29 \cdot 75 \\ 47 \cdot 63 \end{array}$ | $\begin{aligned} & +1 \cdot 31 \\ & -2.95 \\ & -4.46 \end{aligned}$ | Slightly southern Slightly southern Slightly southern |
| Group 6 . | $75 \quad 33$ | - | 2245 | $4 \cdot 63$ | 2245 | $6 \cdot 66$ | $-2.03$ |  |
| Gurária Aramlia | $\begin{array}{ll} 76 & 7 \\ 75 & 2 \end{array}$ | $\begin{aligned} & 1360 \\ & 1532 \end{aligned}$ | $\begin{array}{ll} 24 & 25 \\ 24 & 25 \end{array}$ | $\begin{array}{r} 31 \cdot 98 \\ 2 \cdot 66 \end{array}$ | $\begin{array}{ll} 24 & 25 \\ 24 & 25 \end{array}$ | $\begin{array}{r} 32 \cdot 30 \\ 7 \cdot 11 \end{array}$ | $\begin{aligned} & -0.32 \\ & -4.45 \end{aligned}$ | Inappreciable Slightly northern |
| Group 7 | $75 \quad 35$ | -• | $24 \quad 25$ | $17 \cdot 32$ | $24 \quad 25$ | 19.71 | -2.39 |  |
| Kánkra <br> Khamor . | $\begin{array}{ll} 76 & 10 \\ 74 & 50 \end{array}$ | $\begin{aligned} & 1652 \\ & 1393 \end{aligned}$ | $\begin{array}{ll} 25 & 37 \\ 25 & 45 \end{array}$ | $\begin{aligned} & 58 \cdot 75 \\ & 11.00 \end{aligned}$ | $\begin{array}{ll} 25 & 37 \\ 25 & 45 \end{array}$ | $\begin{aligned} & 59.37 \\ & 14.85 \end{aligned}$ | $\begin{aligned} & -0.62 \\ & -3.85 \end{aligned}$ | Inappreciable <br> None |
| Group 8 | $75 \quad 30$ | . | 2541 | 34.87 | 2541 | $37 \cdot 11$ | -2.24 |  |
| Jetgarh <br> Banksho <br> Rewat | $\begin{array}{ll} 74 & 21 \\ 76 & 11 \\ 74 & 19 \end{array}$ | $\begin{aligned} & 1967 \\ & 1870 \\ & 1542 \end{aligned}$ | $\begin{array}{ll} 26 & 18 \\ 26 & 50 \\ 26 & 53 \end{array}$ | $\begin{array}{r} 8.02 \\ 2.37 \\ 54.74 \end{array}$ | $\begin{array}{ll} 26 & 18 \\ 26 & 50 \\ 26 & 53 \end{array}$ | $\begin{array}{r} 6.23 \\ 7.73 \\ 53.82 \end{array}$ | $\begin{array}{r} +1 \cdot 79 \\ -5 \cdot 36 \\ +0 \cdot 92 \end{array}$ | Southern <br> Decided northemı None |
| Group 9 | $74 \quad 57$ | - | $26 \quad 40$ | 41.71 | $26 \quad 40$ | 42.59 | -0.88 |  |


| Station. | Longitude. | Height in feet. | Latitude. |  |  | Eye estimate of direction and magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. | Grodetic. | A -G. |  |
|  | $\begin{array}{rr}\circ \\ 76 & 15 \\ 75 & 4 \\ 75 & 3\end{array}$ | $\begin{array}{r} 20 \text { 200 } \\ 1204 \\ 951 \end{array}$ | ${ }^{\circ} 7$ 52 $59 \cdot 49$ <br> 27 55 30.05 <br> 28 29 38.81 | $\begin{array}{lll}\circ \\ 27 & 52 & 59 \\ 27 & 1 / 31 \\ 28 & 50 & 30 \cdot 39 \\ 28 & 29 & 39 \cdot 11\end{array}$ | +1.18 +0.18 -0.34 -0.30 | None <br> None <br> None |
| Group 10. | $75 \quad 27$ | - | $\begin{array}{lll}28 & 6 & 2.78\end{array}$ | $28 \quad 6 \quad 2.93$ | -0.15 |  |
| $\underset{\text { Rawaipur }}{\text { Ráki . . . . . . . . }}$ | $\begin{array}{ll}76 & 9 \\ 75 & 6\end{array}$ | 785 697 | $\begin{array}{lll}29 & 17 & 20 \cdot 76 \\ 29 & 39 & 13 \cdot 13\end{array}$ | $\begin{array}{lll} 29 & 17 & 21 \cdot 12 \\ 29 & 39 & 13 \cdot 80 \end{array}$ | $\begin{aligned} & -0 \cdot 36 \\ & -0.67 \end{aligned}$ | None None |
| Groap 11. . | $75 \quad 38$ | - | $\begin{array}{lll}29 & 28 & 16.94\end{array}$ | $\begin{array}{lll}29 & 28 & 17 \cdot 46\end{array}$ | -0.52 |  |
| Khimuána Isanpár | $\begin{array}{ll} 75 & 3 \\ 76 & 9 \end{array}$ | $\begin{aligned} & 731 \\ & 874 \end{aligned}$ | $\begin{array}{lll} 30 & 22 & 11 \cdot 74 \\ 30 & 38 & 16 \cdot 03 \end{array}$ | $\begin{array}{lll} 30 & 22 & 14 \cdot 66 \\ 30 & 38 & 1985 \end{array}$ | $\begin{aligned} & -2.92 \\ & -3.82 \end{aligned}$ | None None |
| Group 12. | $75 \quad 36$ | -• | $\begin{array}{lll}30 & 30 & 13\end{array}$ | $\begin{array}{llll}30 & 30 & 17 \cdot 26\end{array}$ | $-3 \cdot 37$ |  |
| Sangatpar . . . . . Amritsar Ahahpur. | $\begin{array}{rr} 75 & 5 \\ 74 & 55 \\ 75 & 8 \end{array}$ | $\begin{aligned} & 779 \\ & 770 \\ & 830 \end{aligned}$ | $\begin{array}{ccc} 31 & 17 & 35 \cdot 42 \\ 31 & 38 & 2 \cdot 53 \\ 32 & 1 & 34 \cdot 23 \end{array}$ | $\begin{array}{rrr} 31 & 17 & 34 \cdot 27 \\ 31 & 37 & 58 \cdot 56 \\ 32 & 1 & 33 \cdot 61 \end{array}$ | $\begin{aligned} & +1 \cdot 15 \\ & +3 \cdot 97 \\ & +0 \cdot 62 \end{aligned}$ | $\left\{\begin{array}{l} \text { None } \\ \text { Possibly distant Himalayan } \end{array}\right.$ |
| Group 13. . | $75 \quad 3$ | . | $31 \quad 39 \quad 4.06$ | $\begin{array}{lll}31 & 39 & 2.15\end{array}$ | +1.91 |  |

Comparisons of Astronomical with Geodetic Latitudes.
Bombay Meridional Triangulation.


Comparisons of Astronomical with Geodetic Latitudes-(continued).
Calcutta-Kurrachee Longitudinal Series, omitting Stations which have already been included in the Meridional Arcs.

| Name of station. | Longitude. | Height in feet above sea. | Latitude. |  |  |  |  | Eye estimate of direction and magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. | Geodetic. |  |  | A-G. |  |
| Calcutta . . . . . . | $88{ }^{3} \quad 24$ | 25 |  | $2{ }^{2}$ | 32 | 54:51 | +1'07 | None |
| Malúncha . . . . . . | 878 | 970 | $\begin{array}{llll}23 & 54 & 29.64\end{array}$ |  | 54 | $28 \cdot 86$ | +0.78 | Uncertain |
| Chendwar . . . . . . | $85 \quad 29$ | 2817 | $\begin{array}{llll}23 & 57 & 16.82\end{array}$ | 23 | 57 | $13 \cdot 59$ | +3.23 | Southerly |
| Rangír . . . . . . | 7928 | 1184 | $\begin{array}{llll}24 & 0 & 19 \cdot 28\end{array}$ | 24 | 0 | $20 \cdot 21$ | $-0.93$ | None |
| Garwáni . . . . . . | 8220 | 2083 | 24.128 .93 | 24 | 1 | $25 \cdot 55$ | +3:38 | Southerly |
| Kurrachee . . . . . . | 67 4 | 35 | $24 \quad 49 \quad 50 \cdot 14$ |  | 49 | 50.09 | $+0.05$ | None |
| Rambagh . . . . . . | 67 3 | 50 | $\begin{array}{llll}24 & 51 & 20 \cdot 58\end{array}$ |  | 51 | 21.28 |  | None |
| Group . . . . . . | 7934 | -• | $24 \quad 1 \quad .5 \cdot 85$ |  | 1 | $4 \cdot 87$ | $+0.98$ |  |

N The station of Hurilaong is omitted from the above list; it shows $10^{\prime \prime} .91$ southerly attraction; it was originally chosen as a latitude station merely because it happened to be the station of origin of the Harilaong meridional series; but it is obviously unsuited for a latitude station, as the country to the North is described as " an extended plain, while the country to the South is exceedingly hilly."

Bombay Longitudinal Series, omitting Stations which have been already included in the Meridional Arcs.

| Name of station. | Longitude. | Height in feet above sea. | Latitude. |  |  | Eye estimate of direction and magnitude of meridional attraction. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. | Geodetic. | A-G. |  |
| Mándvi . . | 73 | 4121 | $\begin{array}{lll}18 & 37 & 48.09\end{array}$ | $\begin{array}{lll}18 & 37 & 50 \prime 9\end{array}$ | -2'86 | Uncertain, possibly southern |
| Dhauleshvar | 7412 | 2939 | $\begin{array}{lll}18 & 25 & 42 \cdot 91\end{array}$ | $\begin{array}{lll}18 & 25 & 41-48\end{array}$ | +1.43 | Southerly |
| Kanheri . | 75 <br> 76 | 2610 | $\begin{array}{llll}18 & 29 & 21.94\end{array}$ | $\begin{array}{lll}18 & 29 & 30 \cdot 59\end{array}$ | -8.65 | Possibly some northern |
| Nitáli | $76 \quad 19$ | 2:89 | $\begin{array}{lll}18 & 17 & 2.80\end{array}$ | $\begin{array}{lll}18 & 17 & 7.00\end{array}$ | -4.20 | None |
| Group . . . . . | $74 \quad 58$ | . | $\begin{array}{lll}18 & 27 & 28.9 \pm\end{array}$ | $\begin{array}{lll}18 & 27 & 32 \cdot 51\end{array}$ | $-3.57$ |  |

Comparisons of Astronomical with Geodetic Latitudes-(concluded).
Bider Longitudinal Series, omitting Stations which have already been included in the Meridional Arcs.


Himalayan Stations and Stations obviously influenced by Himalayan Attraction.

| Name of station. | iongitude. | Height in feet above sea. | Latitude. |  |  |  |  |  | A-G. | $A-G_{c}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Astronomical. |  |  | Geodetic. |  |  |  |  |
| Rámuápur . | $80{ }^{\circ} 31$ | 541 | 28 | 22 | 0:10 | 28 | 22 | $10 \times 88$ | $-10^{\prime \prime} 78$ | $-10^{\prime \prime} 15$ |
| Nojli . . . . . . . . . | 7743 | 929 | 29 | 53 | $14 \cdot 12$ | 29 | 53 | $27 \cdot 60$ | -13.48 | $-12.86$ |
| Dehra Base, East end . . . | 781 | 1958 | 30 |  | 37-29 | 30 | 17 | $7 \cdot 19$ | - 29.90 | -28.90 |
| Dehra Observatory . . . . | 786 | 2289 | 30 | 19 | 19.56 | 30 | 19 | 56.91 | $-37 \cdot 35$ | $-36.35$ |
| Amsot . . . . . | $77 \quad 44$ | 3140 |  |  | 16.02 | 30 | 22 | $44 \cdot 70$ | -28.68 | -27.66 |
| Rajpar . . | 787 | 2900* | 30 | 23 | $9 \cdot 33$ | 30 | 23 | 56.51 | -47.18 | $-46 \cdot 16$ |
| Masscoree . . . . . . | 787 | 6937 |  | 27 | $4 \cdot 24$ | 30 | 27 | $40 \cdot 39$ | $-36 \cdot 15$ | $-35 \cdot 12$ |
| Banog . . . . . . . | 78 3 | 7433 | 30 | 28 | $4 \cdot 18$ | 30 | 28 | 36.75 | $-32.57$ | $-31 \cdot 54$ |
| Murree . . . . . . . . | $73 \quad 27$ | 7000* |  |  | $37 \cdot 35$ | 33 | 54 | $57 \cdot 19$ | $-19.84$ | $-18.00$ |
| Gogipatri . . . . . . . | 74.48 | 7752 | 33 | 51 | 46.93 | 33 | 51 | $43 \cdot 73$ | a +3.20 +14.93 | $\begin{array}{r}\text { a } \\ +5.03 \\ \hline 16.79\end{array}$ |
| Poshkar . . . . . . | 7432 | 8323 |  | 2 | $3 \cdot 78$ | 34 | 1 | $48 \cdot 85$ | +14.93 | $+16.79$ |

[^3]
## The Longitudinal Arcs.

These arcs lie between certain stations of the triangulation which have been put into connection with the wires of the great electric telegraph lines, thus enabling comparisons to be made between the geodetic and the astronomical differences of longitude.

Two transit instruments with collimators, two astronomical clocks, and two chronographs, with such other electric appliances as were necessary, were obtained from England for the operations. Colonel Strange-a retired officer of the Survey who was then in charge of the Mathematical Instrument Department of che India Office - was entrusted with the provision of the apparatus; and he employed Messrs. Cooke and Sons of York for the construction of the transit instruments and their collimators, Messrs. Frodsham of London for the clocks, and Messrs. Fichen and Hardy of Paris for the chronographs and electric appliances. Full descriptions and drawings of the instruments are given in vol. 9 of the Accounts of the Operations, \&c. ; a brief description is all that is needed here.

The transit instruments were constructed, one of gun-metal, the other of aluminiumbronze ; their telescopes have a focal length of nearly 6 feet and are furnished with 5 -inch object glasses. For portability they are constructed in three pieces, viz., the axis, the object end and the eye end, which travel in separate cases. The axis has a central cube having $9 \frac{1}{2}$-inch sides with conical arms tapering outwards from $9 \frac{1}{4}$ to 3 inches in diameter, and terminating in enlarged cylindrical shoulders $3 \frac{3}{4}$ inches in diameter and 2 inches wide, into each of which a steel pivot is shrunk which projects 1.9 inches beyond the shoulders; the total length of the axis is 37.3 inches, and the thickness of metal is about 0.37 inch throughout, the cube and cones being cast in one piece. The steel pivots are 1.9 inches in diameter, one having a perforation 0.9 inch in diameter, through which light is passed for the illumination of the wires. The object and eye end tubes of the telescope are attached to the cube of the axis by 12 powerful steel bolts which pass through a projecting flange at the base of each tube and screw into the metal of the cube. The two faces of the cube which are at right angles to the faces carrying the telescope are perforated in their centres by $3 \frac{1}{4}$-inch openings through which the collimators are set on each other. The eye piece is fitted with two diaphragms, one containing a single vertical and two horizontal wires, the other a set of 25 vertical wires arranged in groups of 5 each for the observation of startransits, the former fixed, but the latter movable by a micrometer screw. Two kinds of illumination are provided, which give the observer a choice of either dark wires in a bright field or bright wires in a dark field.

The pivots of the axis rest on Y's in gun-metal beds, one of which has footscrews for the adjustment to horizontality, while the other has a provision for the adjustment in azimuth. These pivot beds rest on iron foundation plates, which are placed on the
heads of the masonry supporting piers ; the piers are sunk to a considerable depth in the ground and are carefully insulated from the tread of the observer.

The levelling is performed with a Bohnenberger eyepiece, over a trough of mercury in the nadir ; no spirit-levels are employed.

The two collimators are telescopes with focal lengths of 24 inches and $2 \frac{1}{2}$-inch object glasses; they are set up one to the north the other to the south of the transit instrument, usually at a distance of about 15 feet, on insulated masonry pillars.

The chronographs drive a revolving drum by clockwork which is furnished with a very exact regulator designed by M. Foucault. The drum is about 12 inches wide and 36 inches in circumference ; it carries a paper on which the seconds of the clock time and the moments of the observed transits are impressed electrically by a pair of recording styles; it revolves once in two minutes and is wholly traversed by the styles in three hours; a second of time corresponds to three-tenths of an inch linear space on the drum.

The astronomical clocks are furnished with mercurial pendulums, and carry a wheel with 60 teeth, which revolves once in a minute and breaks the electric current at every second, the tooth opposite $0^{8}$ on the dial being cut off to mark the commencement of each minute.

The observations were always taken in a portable observatory tent made of canvas and provided with shutters and curtains admitting of a meridional aperture from the horizon to the zenith; as it was only sufficiently large to hold the transit telescope, the chronograph and clock were provided for in a room in a neighbouring building.

The stations were almost invariably situated in the neighbourhood of a Government Telegraphic Office, and were invariably connected with the nearest stations of the Great Triangulation, so that their geodetic latitudes and longitudes may be accepted with confidence.

The instruments were received in India in 1872, and were at first employed at Bangalore and Madras, which were nearly on the same parallel, and were connected by a railroad, so that the arc between the two places was a very suitable one for testing the capabilities of the instrumental equipment. The initial operations were far from satisfactory; one of the transit instruments, No. 2, showed discordances in the determination of the "constant for collimation," which were eventually traced to a defect in construction. This was duly rectified by the mathematical instrument maker at Madras, and for the three next field seasons the instruments were apparently in excellent working condition.

During the observations at each pair of stations, the same stars were observed at both stations, at their transits across the two meridians, thus eliminating error of star's place. The chronograph at each station received its time signals from the clocks at both stations in alternate batches, and the retardation of the electric current in its passage along the wire was thus measured very exactly.

Great care was taken throughout the operations to make them self-verificatory. The stations were selected in such a manner as to form a network of great triangles over the entire face of the country, and observations were taken on every side of each triangle ; thus in every case a triangular circuit was completed, the closing error of which was a valuable indication of the general accuracy of the operations.

In the field season of 1875-76 six arcs were measured; in 1876-77, three arcs, and the differences in longitude between Bombay, Aden and Suez, to complete a connection of India with England through the determination of Greenwich to Suez for the Transit of Venus Expedition in 1874. The Afghan War and other circumstances led to the suspension of operations until 1880-81, when they were resumed and eight arcs were measured. The operations of these three field seasons complete seven circuits, with errors ranging from $\cdot 007^{s}$ to $\cdot 083^{s}$, with the average value $035^{s}$, which were as small as could be reasonably expected. (See page 260 of vol. 9.)

In 1881-82 seven arcs were measured, completing three circuits, of which the closing errors were ${ }^{\circ} 046^{8},{ }^{\circ} 291^{8}$, and $\cdot 556^{8}$, two of them so large as to show that something had gone wrong. The collimation determinations showed that Transit Instrument No. 2 had again become unsatisfactory, as it was in 1874-75. It was rectified in the Mathematical Instrument Department at Calcutta, and operations were resumed in the two following field seasons, during which three of the arcs of 1881-82 were re-measured, and six new circuits were completed with small closing errors ranging from $\cdot 027^{\circ}$ to $\cdot 081^{s}$.

Thus it seemed that the faulty transit instrument had been put into a satisfactory working condition; but as some uneasiness was still felt regarding the instruments, they were returned to the makers in England for examination and rectification ; afterwards they were carefully tested at the Greenwich Observatory, and were pronounced to be in perfect condition.

Operations were resumed in 1885-86; the instruments worked very satisfactorily, but again large circuit errors were obtained, and similar results were met with up to the close of the operations in 1891-92. The published annual reports of the Survey show what anxious attention was paid to these mysterious circuit errors, and what a variety of possible causes of error were examined from time to time with a view to ascertaining the actual cause. Happily it was eventually discovered, as will now be shown.

The transit instruments had invariably been set up between two collimating telescopes for the determination of their collimation constants by Gauss's well-known method. They were also invariably reversed on their Y's at every station, so that half the observations were taken with the illuminated pivot to the east, and half with it to the west; at first there were not more than one or two reversals during the whole of the observations at a station, but latterly the number was much increased, and a reversal might be made in the course of a single night's work Now, if the collimation error is really constant and invariable, reversal of pivot removes all
necessity for determining its magnitude, as the error it produces in a single observation of a star is eliminated in the mean of a pair of observations to the same star in the two instrumental positions. But values of the error had always been determined from the very first by observations to the two collimators; a value was found for each position of the transit instrument, and applied to all stars taken in that position. Very frequently the values found in the two positions were closely identical, but occasionally they differed materially, and when this took place it was attributed partly to errors of observation, but mainly to actual changes in the collimation angle on reversal. The process of reversal was not an easy matter ; the weight of the instrument was considerable, 137 lbs.; there was no apparatus for raising it vertically and dropping it down again after reversal; thus, the following procedure had to be adopted for reversing:-the telescope was laid horizontally, two long poles were attached to the cube of the transit axis, the instrument was then raised by four men who walked round the supporting pillars with it on their shoulders and then lowered it into position, the observer standing at one end and his assistant at the opposite end to guide the pivots exactly into the Y's.

When large discrepancies were met with they were for a long time assumed to be due in some way to this process of reversal. But the largest of all occurred at Deesa, in 1885, only a few weeks after the return of the instruments from the hands of the makers in England; there the instrument was reversed every night for six nights, and the collimation error was determined in each position ; those in the same position agreed with each other very closely, within an extreme range of 8 of a division of the micrometer, while those in opposite position differed by as much as 8.3 divisions, a quantity far exceeding the possible errors of observation. It was clear, therefore, that the collimation angle had been constant, and that there was some error in its determination. This led Captain Burrard to infer that the observations should be reduced with the mean collimation constant for the two instrumental positions instead of the single constant for each position, and when this was done the circuit errors of large magnitude all disappeared, as will be seen from the following statement.*

The number of circuit errors exceeding $50^{\circ}$ was reduced from . . 4 to 0

| $"$ | $"$ | between $\cdot 50$ and $\cdot 30^{8}$ | was reduced from | 2, | 0 |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $"$ | $"$ | $"$ | $\cdot 30$ | $"$ | $\cdot 20$ | $"$ | , | 5 |

Clearly, therefore, the fault lay in the determination of the collimation constant.

- See Annual Report for 1889-90, p. 71.

The object glasses of the transit telescopes were 5 inches in diameter, those of the collinators were $2 \frac{1}{2}$ inches in diameter, and the apertures in the cube of the transit axis through which the collimators were pointed at each other were 34 inches in diameter ; this left an annulus of $\frac{8}{8}$ inch between the rims of the object glasses of the collimators and the rim of the aperture when the collimators were placed exactly in line with the centre of the transit instrument. They may, however, be placed an inch or more to either side without sensibly affecting the operation of laying their wires on each other, notwithstanding that the displacement causes considerable portions of their object glasses to be cut out of operation by the intermediate cube of the transit instrument. More or less displacement had frequently occurred in practice, as the necessity for exact central adjustment had never impressed itself on the observers, and the alignment was always considered sufficiently good if clear vision of the collimator wires was obtained when they were being set on each other. Displacement would act in two ways; it would bring different parts of the object glass of a transit instrument into play before and after reversal of pivots, and it would cut off the same parts of the object glasses of the collimators throughout the operations; in both cases error would be caused if the object glasses were not truly homogeneous throughout and with truly spherical surfaces. All the object glasses were therefore tested. The transit instruments were set on a suitable distant object, which was observed with the full field of the glass, and then with the upper, lower, eastern, and western half-field in succession ; and the experiments showed that the object glass of transit instrument No. 1 was perfect, and that of No. 2 nearly perfect. Then the collimators were set on each other, and similar observations were taken with the full field and the half-fields; it was at once found that their object glasses were very imperfect, showing differences between the east and the west halves, which fully accounted for the discordances which had been met with in the determination of the collimation constant. The experiments are fully detailed at pp. 379 to 387 of vol. 15.

Thus, this very perplexing mystery was at length traced to its source, and with the happy result that nine out of ten years' operations gave precise and satisfactory results by the employment of the mean collimation constant, instead of the positional constants which had been previously employed. The work of 1881-82 had to be rejected, because the collimation observations, given in volume 10 , show that the tube of the telescope of No. 2 had been shaky, and consequently inefficient, throughout the field seasnn, necessitating the rejection of the whole of the operations during that field season.

Thus, finally, fifty-five longitudinal arcs were obtained. They present thirty-one circuits, of which the errors are known, and thus give so many equations of condition for solution to determine the most probable values of the errors of the individual arcs. The equations were solved in the usual manuer by the method of minimum squares, giving equal weights to all the arcs.
mDCccexer.-A.
5 K

$$
\bullet
$$



The following tables give, first, a list of the longitude stations, with their latitudes, longitudes, and heights ; secondly, a list of the arcs, arranged from west to east, with their distinguishing numbers and observed values; thirdly, a list of the circuits, with their errors; and, fourthly, a list of the most probable corrections to the individual arcs, as determined by the process of calculation just described. These are followed by Table V., which gives a comparison of the Astronomical with the Geodetic values of the Arcs.

Table I.—Geodetic Elements of the Longitude Stations.

|  | Latitude. | Longitude. | Height, in feet. |
| :---: | :---: | :---: | :---: |
| Agra |  |  | 550 |
| Akyab | $20 \quad 814.29$ | 925616.57 | 20 |
| Amritsar | 313758.72 | $745450 \cdot 63$ | 770 |
| Bangalore. | $13 \quad 041.29$ | $773727 \cdot 37$ | 3110 |
| Bellary. . | $15 \quad 8 \quad 33.06$ | 76586.76 | 1500 |
| Bolarum | $173013 \cdot 41$ | 7833 38.30 | 1600 |
| Bombay (Colaba) | $185349 \cdot 49$ | 725116.28 | 40 |
| Calcatta | 223254.99 | 882356.28 | 20 |
| Chittagong | $\begin{array}{llllll}22 & 20 & 19.53\end{array}$ | 915243.03 | 90 |
| Deesa . - | $2415 \quad 29 \cdot 35$ | 721332.03 | 440 |
| Dehra Dan | $301929 \cdot 13$ | $\begin{array}{ll}78 & 5 \\ 49 & 39\end{array}$ | 2200 |
| Fyzubad | $264640 \cdot 66$ | $821035 \cdot 33$ | 830 |
| Jabalpar | $231010 \cdot 10$ | $7959 \quad 29 \cdot 79$ | 1350 |
| Jalpaigari. | $263117 \cdot 39$ | $884639 \cdot 95$ | 280 |
| Kalianpur. | $\begin{array}{lllll}24 & 710.91\end{array}$ | 774144.75 | 1765 |
| Karrachee | $2451 \quad 2 \cdot 44$ | $67 \quad 3 \quad 20 \cdot 40$ | 50 |
| Madras. | $\begin{array}{llll}13 & 4 & 3.75\end{array}$ | $801721 \cdot 51$ | 20 |
| Mangalore | 125214.14 | $\begin{array}{lllllllllll}74 & 53 & 989\end{array}$ | 175 |
| Mooltan | 301058.70 | $712854 \cdot 57$ | 420 |
| Moulmein . | 162956.49 | 97408.05 | 90 |
| Nagarkoil . | $81125 \cdot 30$ | 77283074 | 110 |
| Peshawur . | $\begin{array}{llll}34 & 0 & 17.51\end{array}$ | $7135 \quad 27 \cdot 45$ | 1100 |
| Prome | $184915 \cdot 12$ | $9515 \quad 24.91$ | 100 |
| Quetta | $301157 \cdot 29$ | $67 \quad 258.87$ | 5500 |
| Waltair | $1743 \quad 2931$ | $832130 \cdot 70$ | 500 |

The longitudes are from Greenwich, and they require a constant correction, which is usually taken as $-2^{\prime} 30^{\prime \prime}$, to reduce them to the latest determination of the Madras Observatory relatively to Greenwich.

Table II.-List of Arcs with their Distinguishing Numbers and Observed Values.

| Name of arc. | No. | Observed value. | Name of arc. | No. | Observed value. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mooltan-Quetta | (1) | $\begin{array}{cc} \mathrm{m} & \mathrm{~s} \cdot \\ 17 & 43 \cdot 499 \end{array}$ | Bellary-Bombay . | (29) |  |
| Kurrachee-Quetta | (2) | $\begin{array}{lll}0 & 1.603\end{array}$ | Bangalore-Bellary | (30) | 2 37-230 |
| Mooltan-Kurrachee . | (3) | 1741.976 | Bellary-Mangalore | (31) | $819 \cdot 653$ |
| Agra-Deesa | (4) | 23 20•370 | Mangalore-Bombay | (32) | $8 \quad 7 \cdot 273$ |
| Deesa-Kurrachee | (5) | $2040 \cdot 529$ | Bangalore-Nagarkoil . | (33) | $035 \cdot 708$ |
| Deesa-Mooltan | (6) | 258.582 | Nagarkoil-Mangalore | (34) | $1021 \cdot \mathrm{ral}$ |
| Agra-Karrachee . | (7) | $44 \quad 0.992$ | Madras-Nagarkoil | (35) | 1115.006 |
| Agra -Mooltan | (8) | 2619.053 | Madras-Bangalore | (36) | 1039.331 |
| Amritsar-Mooltan | (9) | 13 44:281 | Madras-Mangalore | (37) | $2136 \cdot 129$ |
| Peshawar-Mooltan | (10) | $027 \cdot 483$ | Madras-Bellary | (38) | $13 \quad 16 \cdot 567$ |
| Amritsar-Peshawar . | (11) | 1316.776 | Madras-Bolarum. | (39) | $654 \cdot 615$ |
| Agra-Amritsar | (12) | 1234.725 | Waltair-Madras | (40) | 1216.868 |
| Dehra Dun-Agra | (13) | $0 \quad 77233$ | Waltair-Bolarum | (41) | $1911 \cdot 525$ |
| Dehra Dan-Amritsar | (14) | 1241.995 | Waltair-Jnbbulpore | (42) | 1328.501 |
| Fyzabad-Agra | (15) | 1627.995 | Calcutta-Waltair | (43) | $\begin{array}{ll}20 & 9 \cdot 194\end{array}$ |
| Fyzabad-Dehra Dan | (16) | $1620 \cdot 704$ | Calcutta-Jabbulpore | (44) | 33 37.702 |
| Fyzabad-Jubbulpore | (17) | 845.040 | Calcatta-Fyzabad | (45) | 2452.661 |
| Jabbalpore-Agra . | (18) | 74.3026 | Jalpaigari-Calcutta | (46) | $130 \cdot 290$ |
| Jubbulpore-Deesa | (19) | $31 \quad 3.393$ | Jalpaiguri-Fyzabad | (47) | 2624.986 |
| Jubbulpore-Bombay | (20) | $28: 31.816$ | Chittagong-Calcutta | (48) | 13 55.145 |
| Jabbulpore-Kalianpur | (21) | $910 \cdot 323$ | Chittagong-Jalpaiguri . | (49) | 1224816 |
| Agra-Kalianpar . | (22) | $127 \cdot 319$ | Akyab-Chittagong | (50) | $414 \cdot 252$ |
| Kalianpar-Bombay | (23) | 19 21-441 | Akyab-Calcntta | (51) | $18 \quad 9 \cdot 395$ |
| Bombay-Kurrachee . | (24) | 23 12-215 | Prome-Akyab. | (32) | $916 \cdot 262$ |
| Bombay-Deesa | (25) | 231.644 | Prome-Chittagong | (53) | 13 30-472 |
| Jubbulpore-Bolarum | (26) | 542.935 | Moulmein-Prome | (54) | $938 \cdot 758$ |
| Bolarum-Bombay | (27) | 2248.785 | Moulmein-Akyab | (55) | 18 54:974 |
| Bolarum-Bellary . | (28) | 621.943 |  |  |  |

5 K 2

Table III. Circuit Errors.

| I |  |
| :---: | :---: |
| II | $(4)+(5)-(7)=-.093$ |
| III | (4) $+(6)-(8)=-101$ |
| IV | $(6)+(3)-(5)=+029$ |
| V | $(12)+(9)-(8)=-047$ |
| VI | (11) $+(10)-(9)=-022$ |
| VII | (13) $+(12)-(14)=-037$ |
| VIII | (16) $+(13)-(15)=-058$ |
| IX | $(17)+(18)-(15)=+071$ |
| $\mathbf{X}$ | $(18)+(22)-(21)=+022$ |
| XI | (18) $+(4)-(19)=+.003$ |
| XII | $(21)+(23)-(20)=-052$ |
| XIII | (20) $+(25)-(19)=+.067$ |
| XIV | $(25)+(5)-(24)=-042$ |
| XV | $(26)+(27)-(20)=-.096$ |
| XVI | $(28)+(29)-(27)=+0.25$ |
| XVII | $(31)+(32)-(29)=+.059$ |
| XVIII | $(35)+(34)-(37)=+018$ |
| XIX | $(38)+(31)-(37)=+091$ |
| $\mathbf{X X}$ | $(36)+(30)-(38)=-006$ |
| XXI | $(36)+(33)-(35)=+\cdot 033$ |
| XXII | $(39)+(28)-(38)=-009$ |
| XXIII | $(40)+(39)-(41)=-.042$ |
| XXIV | $(42)+(26)-(41)=-.089$ |
| XXV | $(43)+(42)-(44)=-.007$ |
| XXVI | $(4.5)+(17)-(44)=-.001$ |
| XXV1I | (46) + (45) -- (47) $=-.035$ |
| XXVIII | $(49)+(46)-(48)=-0399$ |
| XXIX | $(52)+(50)-(53)=+.042$ |
| $\mathbf{X X X}$ | $(54)+(52)-(55)=+.046$ |
| XXXI | $(50)+(48)-(51)=+\cdot 002$ |


| Arc. | Correction. | Arc. | Correction. |
| :---: | :---: | :---: | :---: |
| (1) | 8. $+\quad .029$ | (29) | s. + +004 |
| (2) | -. 029 | (30) | -. 003 |
| (3) | -. 022 | (31) | -. 041 |
| (4) | +.045 | (32) | -. 014 |
| (5) | +.030 | (33) | -. 010 |
| (6) | +.023 | (34) | .000 |
| ( 7 ) | -. 018 | (35) | +.010 |
| (8) | -. 033 | (36) | -. 013 |
| (9) | +.004 | (37) | +.028 |
| (10) | +.013 | (38) | -. 022 |
| (11) | +.013 | (39) | -000 |
| (12) | +.010 | (40) | +.006 |
| (13) | +.032 | (41) | -036 |
| (14) | +.005 | (42) | +.016 |
| (15) | +.012 | (43) | -.016 |
| (16) | +.038 | (44) | -007 |
| (17) | -. 028 | (45) | +022 |
| (18) | -. 031 | (46) | +015 |
| (19) | +.017 | (47) | +.002 |
| (20) | -. 043 | (48) | -.007 |
| (21) | -000 | (49) | +.017 |
| (22) | +.009 | (50) | -. 004 |
| (23) | +009 | (51) | -.009 |
| (24) | -. 019 | (52) | -.024 |
| (25) | $-\cdot 007$ | (53) | +.014 |
| (26) | $+.037$ | (54) | -.011 |
| (27) $(28)$ | +.016 -.013 | (55) | +.011 |

The following table, No. V., gives a comparison of the final astronomical values of the longitudinal arcs-as obtained after applying the corrections in Table IV. for the errors of the circuits-with the originally computed geodetic values in terms of the adopted elements of the Earth's figure, and with those values corrected to correspond with Clarke's latest elements of the figure by the formulæ already given on p. 765. The two sets of results are given, both in time and in arc, in the columns headed $A-G$, and $A-G_{c}$. The data under $A-G$ show the extent to which the results are affected both by errors in the geodetic determinations arising from the adopted elements of the Earth's Gigure and by errors in the astronomical determinations arising from local deflections of the plumb-line; the data under $A-G_{c}$ may be regarded as well nigh free from the first and showing the second only. The algebraical sum of the quantities $A-G$ is $145^{\prime \prime} \cdot 88$, while that of the quantities $A-G_{c}$ is only $29^{\prime \prime} \cdot 23$, showing that the latter quantities are most free from constant error, such as would be caused by employing erroneous axes in the geodetic calculations. Obviously, therefore, it was very necessary to separate the two classes of error. Thus it is to the column $A-G_{c}$ only that we have to look hereafter for an indication of the probable amounts of the local deflections of the plumb-line on the prime vertical to be met with in the longitudinal arcs.

Table V.-Comparison of Astronomical with Geodetic Values.

|  | Name of arc. | Astronomical | Seconds of geodetic value. |  | A - $\boldsymbol{G}$. |  | A - $\mathbf{G}_{\text {c }}$. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Original. | Corrected. |  |  |  |  |
|  |  | A. | G. | $\mathrm{G}_{\text {c }}$. | In time. | In arc. | In time. | In aro. |
|  | M | $\begin{gathered} \mathrm{m} .8 . \\ 17 \\ 43 \cdot 528 \end{gathered}$ | $\begin{gathered} \text { s. } \\ 43 \cdot 714 \end{gathered}$ | $43 \cdot 526$ | $\stackrel{\text { s. }}{-0.186}$ | - 2.79 | $\begin{array}{r} \text { s. } \\ +0.002 \end{array}$ | 1003 $+\quad 0.03$ |
| (2) | Kurrachee-Quetta | $0{ }^{1} 1 \cdot 574$ | 1-436 | 1.426 | +0.138 | + 2.07 | $+0.138$ | + 2.07 |
| (3) | Moultan-Kurrachee | 1741.954 | $42 \cdot 278$ | 42.093 | -0.324 | - 4.86 | -0.139 | - 2.09 |
| (4) | Agra-Deesa | 2320.415 | 19.803 | $19 \cdot 562$ | +0.612 | $+9.18$ | +0.853 | +12.80 |
| (5) | Deesa-Kurrachee | $2040 \cdot 559$ | $40 \cdot 775$ | $40 \cdot 563$ | -0.216 | - 3.24 | -0.004 | - 0.06 |
| (6) | Deesa-Moultan | 258.605 | 58.497 | $58 \cdot 466$ | +0.108 | + 1.62 | +0.139 | + 2.09 |
| (7) | Agra-Kurrachee | $44 \quad 0.974$ | 0.578 | $0 \cdot 123$ | +0.396 | + $5 \cdot 94$ | +0.851 | +12.77 |
| (8) | Agra-Mooltan . | 2619.020 | $18 \cdot 300$ | 18.023 | +0.720 | +10.80 | +0.997 | +14.95 |
| (9) | Amritsar-Moultan | 1344.285 | 43.737 | $43 \cdot 590$ | +0.548 | + 8.22 | +0.695 | $+10.43$ |
| (10) | Peshawar-Mooltan | 0 $27 \cdot 496$ | $26 \cdot 192$ | 26.187 | +1.304 | +19.56 | +1.309 | +19.64 |
| (11) | Amritsar-Peshawar | 1316.789 | 17.545 | $17 \cdot 401$ | -0.756 | $-11 \cdot 34$ | -0.612 | - $9 \cdot 18$ |
| (12) | Agra-Amritsar | 1234.735 | 34.563 | 34.430 | +0.172 | + $2 \cdot 58$ | +0.305 | + 4.58 |
| (13) | Dehra Dun-Agra. | 0 7 | $9 \cdot 348$ | 9.346 | -2.083 | -31.25 | -2.081 | $-31 \cdot 22$ |
| (14) | Dehra Dan-Amritsar | $1242 \cdot 000$ | 43.911 | 43.775 | -1.911 | -28.67 | -1.775 | $-26.63$ |
| (15) | Fyzabad-Agra. | 1628.0017 | $28 \cdot 417$ | 28.246 | -0.410 | -6.15 | $-0.239$ | -3.59 |
| (16) | Fyzabad-Dehra Duı | $1620 \cdot 742$ | 19.069 | 18.897 | +1.673 | +25.10 | +1.845 | $+27 \cdot 68$ |
| (17) | Fyzabad--Jubbulpore | 845.012 | 44.369 | 44:279 | +0.643 | + 9.65 | +0.733 | $+11.00$ |
| (18) | Jubbulpore-Agra. | $742 \cdot 995$ | 44.048 | 43.968 | -1.053 | -15.80 | -0.973 | $-14.60$ |
| (19) | Jubbulpore-Deesa | 31 3.410 | 3.851 | $3 \cdot 535$ | -0.441 | -6.62 | -0.125 | - 1.88 |
| (20) | Jubbulpore--Bombay . | 2831.773 | 32.901 | 32.615 | -1.128 | -16.92 | -0.842 | $-12.63$ |
| (21) | Jabbulpore-Kalianpar | $\begin{array}{lll}9 & 10 \cdot 323\end{array}$ | 11.003 | $10 \cdot 909$ | -0.680 | $-10 \cdot 20$ | -0.586 | -8.79 |
| (22) | Agra-Kalianpur . | 127.328 | 26.955 | 26.940 | +0.373 | + 5.60 | +0.388 | + 5.82 |
| (23) | Kalianpar-Bombay | 19 21-450 | 21.898 | $21 \cdot 703$ | -0.448 | + 6.72 | $-0.253$ | - 380 |
| (24) | Bombay-Kurrachee | 23 12196 | 11.725 | $11 \cdot 491$ | $+0.471$ | + 7.07 | +0.705 | +10.58 |
| (25) | Bombay-Deesa | 231.637 | 30.950 | $30 \cdot 925$ | +0.687 | +10.31 | +0.712 | - $+10 \cdot 68$ |
| (26) | Jubbulpore-Bolarum | 542.972 | $43 \cdot 422$ | $43 \cdot 365$ | -0.450 | -6.75 | -0.393 | - 5.90 |
| (27) | Bolarum-Bombay | $2248 \cdot 801$ | $49 \cdot 479$ | $49 \cdot 253$ | -0.678 | $-10.17$ | $-0.452$ | -6.78 |
| (28) | Bolarum-Bellary . | $6 \quad 21 \cdot 930$ | $22 \cdot 114$ | $22 \cdot 052$ | -0.184 | - 2.76 | -0.122 | - 1.83 |
| (29) | Bellary-Bombay . | 1696.871 | 27.365 | 27.203 | -0.494 | - 7.41 | $-0.332$ | - 4.98 |
| (30) | Bangalore-Bellary | ${ }_{2}^{2} 37 \cdot 227$ | 37.374 | 37.349 | -0.147 | - 2.21 | -0.122 | - 1.83 |
| (31) | Bellary-Mangalore | 819.612 | $19 \cdot 791$ | $19 \cdot 710$ | -0.179 | - 2.69 | -0.098 | - 1.47 |
| (32) | Mangalore-Bombay | 887.259 | 7.574 | $7 \cdot 495$ | -0.315 | - 4.73 | -0.236 | - 3.54 |
| (33) | Bangalore-Nagarkoil | 035.698 | 35.775 | $35 \cdot 769$ | -0.077 | $-1.16$ | -0.071 | - 1.07 |
| (34) | Nagarkoil-Mangalore | $1021 \cdot 141$ | 21.390 | 21-291 | -0.249 | - 3.74 | -0.150 | - 2.25 |
| (35) | Madras-Nagarkoil | $1115 \cdot 016$ | 15.385 | 15.278 | -0.369 | - $5 \cdot 54$ | -0.262 | - 3.93 |
| (36) | Madras-Bangalore | $1039 \cdot 318$ | 39.610 | 39-507 | -0.292 | - 4.38 | -0.189 | - 2.84 |
| (37) | Madras-Mangalore | $2136 \cdot 157$ | 36.775 | 36.567 | -0.618 | - 9.27 | -0.410 | $-6.15$ |
| (38) | Madras-Bellary | 1316.545 | 16.984 | 16.855 | -0.439 | -6.59 | $-0.310$ | - 4.65 |
| (39) | Madras-Bolarum | 654.615 | 54.870 | 54.803 | -0.255 | -3.83 | -0.188 | - 2.82 |
| (40) | Waltair-Madras | 1216.874 | 16.612 | $16 \cdot 493$ | +0.262 | +3.93 | +0.381 | + 5.73 |
| (41) | Waltair-Bolarnm. | $19 \quad 11 \cdot 489$ | 11.482 | 11.293 | +0.007 | + 0.11 | +0.196 | + 2.94 |
| (42) | Waltair-Jubbulpore | 13 28.517 | 28.060 | 27.926 | +0.457 | +6.86 | +0.591 | + 8.87 |
| (43) | Calcutta-Waltair . | $20 \quad 9 \cdot 178$ | 9.684 | $9 \cdot 483$ | -0.506 | - 7.59 | -0.305 | - 4.58 |
| (44) | Calcutta-Jubbulpore | 3337.695 | 37.744 | $37 \cdot 403$ | -0.049 | - 0.74 | +0.292 | + 4.38 |
| (45) | Calcutta-Fyzabad | $2452 \cdot 683$ | 53.375 | 53.120 | -0.692 | -10.38 | -0.43? | - 6.56 |
| (46) | Jalpaiguri-Calcutta | $130 \cdot 305$ | 30.933 | 30.917 | -0.628 | - 9.42 | -0.612 | - $9 \cdot 18$ |
| (47) | Jalpaiguri-Fyzabad | ${ }^{26} 222 \cdot 988$ | 24.308 | 24.034 | $-1320$ | $-19.80$ | -1.046 | -15.69 |
| (48) | Chittagong-Calcutta | $1355 \cdot 138$ | $55 \cdot 195$ | 55.054 | -0.057 | - 0.86 | $+0.084$ | +1.26 |
| (49) | Chittagong-Jalpaiguri | 1224.833 | $24 \cdot 262$ | 24.135 | +0.571 | +8.57 | +0.698 | +10.47 |
| (50) | Akyab-Chittagong | 414.248 | 14.236 | 14.193 | $+0.012$ | + 0.18 | +0.055 | + 0.83 |
| (51) | Akjab-Calcutta. | $\begin{array}{ll}18 & 9 \cdot 386\end{array}$ | $9 \cdot 431$ | $9 \cdot 249$ | -0.045 | -0.68 | +0.137 | + 2.06 |
| (52) | Prome-Akyab | $916 \cdot 238$ | 16.556 | $16 \cdot 464$ | -0.318 | - 4.77 | -0.226 | - 3.39 |
| (53) | Prome-Chittagong | $1330 \cdot 486$ | $30 \cdot 792$ | $30 \cdot 657$ | $-0.306$ | - 4.59 | -0.171 | - 2.57 |
| (54) | Moulmein-Prome. | $\begin{array}{ll}9 & 38.747\end{array}$ | 38.876 | 38.781 | -0.129 | - 1.94 | -0.034 | - 0.51 |
| (55) | Moulmein-Akyab | 18 54:985 | $55 \cdot 432$ | 55.245 | -0.447 | $-6.71$ | -0.260 | $-3.90$ |

## On Local Deflections of the Plumb-line.

The stations of a triangulation are not always well adapted as points for the determination of astronomical latitudes and longitudes, for they are necessarily situated on the most commanding positions that the ground affords, and when these positions are on high mountain ranges, or, indeed, on ground of moderate elevation but with an obvious preponderance of matter in one direction, there is an obvious tendency for the plumb-line to be deflected. But it unfortunately happens that an absolute freedom from apparent causes of disturbance is not always accompanied by an absence of disturbance. From the time when latitude observations taken on the plains of Moscow showed large deflections of the plumb-line which were caused, not by visible irregularities on the surface of the ground, but by variations of density in the hidden masses below the surface, similar results must have been met with in all geodetic surveys; they abound in India, and they have recently been exemplified on a large scale in Europe by Colonel Von Sterneck's pendulum investigations of variations of density under the surface of the earth. Thus, in this Survey, it not unfrequently happens that where the observers have been led by the aspects of the ground to anticipate deflection to have taken place in one direction, the observations show that it has really occurred in the opposite direction. Wherever disturbance takes place, the cause may be either an excess of matter, visible or invisible, in one direction, or a deficiency of matter, visible or invisible, in the opposite direction.

The Himalayan mountain ranges have always been regarded as a very disturbing element in Indian geodesy. In fixing on the station of Kaliana, as the northern extremity of the Great Arc, Everest says: "It has not been deemed prudent to approach too near to the mighty mountain masses of the Himalayan range; and rather than that I preferred to shorten the northern section by selecting Kaliana as the station for observing amplitudes, though it is 69 miles more southerly than Banog, and 57 miles to the south even of the base measured in the Dehra Dun."

It now appears that Everest was quite justified in fixing on that station as the northern extremity of the Great Arc. But meanwhile two sets of calculations have been made to show that the plumb-line at Kaliana is largely deflected by Himalayan attraction. J'he first is set forth in a paper-in vol. 17. (1847-48) of the 'Memoirs of the Royal Astronomical Society'-by Captain Shortrede, who assumes that the disturbing effect of the Himalayas may be compared to that of a fixed centre of attraction, and taking for his data the differences between the astronomical and the geodetic latitudes and azimuths at Kaliana and at certain Himalayan stations, finds the direction and distance of a centre of attraction which would produce the observed discrepancies, and calculates the meridional deflection of the plumb-line at Kaliana as something ranging between $11^{\prime \prime}$ and $15^{\prime \prime}$.

The second is an elaborate calculation by Archdeacon Pratt, in a paper "On the Attraction of the Himalayan Mountains and of the Elevated Regions beyond them
upon the Plumb-line in India," which is published in the 'Philosophical Transactions' for 1855, vol. 145. The Archdeacon dissects the masses of which the Himalayas, and the high table lands of 'Tibet, and the regions beyond up to the parallel of $50^{\circ}$, are composed; he calculates their attraction and finds that they cause deflections of the plumb-line in the meridian which amount to $28^{\prime \prime}$ at Kaliana, and to $12^{\prime \prime}$ at Kalianpur, and $7^{\prime \prime}$ at Damargida, the next southern stations on the Great Arc. If these calculations are correct, the astronomical latitude of Kaliana should be corrected to $29^{\circ} 31^{\prime} 16^{\prime \prime} \cdot 0$, of Kalianpur to $24^{\circ} 7^{\prime} 23^{\prime \prime} \cdot 1$, and of Damargardi to $18^{\circ} 3^{\prime} 22^{\prime \prime} \cdot 1$; thus the astronomical amplitudes of the two sections would be $5^{\circ} 23^{\prime} 52^{\prime \prime} \cdot 9$, and $6^{\circ} 4^{\prime} 1^{\prime \prime} \cdot 0$, which exceed the geodetic values by $10^{\prime \prime} \cdot 3$ and $7^{\prime \prime} \cdot 6$, quantities so large that they cannot be accepted as at all probable.

Archdeacon Pratt's paper in the 'Philosophical Transactions' for 1855, is immediately followed by one by Mr.--afterwards Sir George-Airy, Astronomer Royal, who points out that the magnitudes of attractions computed on the theory of gravitation being considerably greater than is necessary to explain the observed discrepancies, is just what we might be prepared to expect when we consider that the crust of the Earth is probably supported by a fluid of greater density than itself, and that great mountain masses such as those to the north of India, necessarily sink into this fluid to a greater depth than the surrounding unelevated portions of the Earth's crust; that this heavy fluid is displaced by light crust, and the positive attraction produced by the elevated masses is diminished by the negative attraction produced by the substitution of light crust for heavy lava. The general conclusion at which Sir George Airy arrived was that, supposing the crust of the Earth to be floating in a state of equilibrium, the real disturbance will, in all cases, be less than that found by computing the effect of the mountains on the law of gravitation; near to the elevated region the part which is to be subtracted from the computed effect is a small proportion of the whole; at a distance the part to be subtracted is so nearly equal to the whole that the remainder may be rejected as insignificant, even in cases where the attraction of the elevated country itself would be considerable.

Sir George Airy's hypothesis, that the Earth is constituted of a light crust floating on fluid lava, has not met with much acceptance. Eminent mathematicians have subsequently brought forward evidence to show that the Earth must be regarded as more probably solid than fluid to its core; and the investigations of Sir William Thomson (Lord Kelvin) suggest that the Earth's mass must possess such a degree of rigidity as to be inconsistent with a crust of moderate thickness. But as yet theories of the internal constitution of the Earth have thrown little light on the subject of the influence of mountain masses on the plumb-line.

The Indian pendulum observations have, however, thrown much light on the subject. They were made at a number of stations on the Great Arc, from Cape Comorin up to Mussooree on the Himalayas, also at a station on one of the high Himalayan plateaus, at various stations on the coast lines, and also on a few islands, one of which
was in the ocean, at a considerable distance from the mainland. The results when reduced to a common temperature, pressure, and origin at the sea-level, and compared with the theoretic vibration numbers computed from the same origin, show the following excesses and defects of observation over theory, as expressed in the daily vibration numbers of a second's pendulum :-*


These results show an excess of density under the sea level, and a deficiency above that level which increases to a very notable magnitude at a bigh altitude on the Himalayas. Thus, therefore, there must be a condensation of the matter of the Earth's crust under ocean beds, and attenuation of the matter under mountains, the crust contracting and condensing wherever it sinks into hollows, and expanding and attenuating when it rises into continents and mountain ranges. Hence it is to be inferred that the matter of which elevations are composed is largely an expansion of the matter in the immediately subjacent strata of the Earth's crust, the masses above and below being closely connected and mutually interdependent; where high elevations exist the strata below having parted with matter are deficient in density; at low elevations there is no appreciable giving off of matter, and the density below is normal ; under the sea there is a contraction of matter, and consequently an increase of density. Now in the application of the theory of gravitation to calculate the attraction of mountain masses on the plumb-line, it is tacitly assumed that the Earth's crust under the sea level is of uniform density throughout, whether lying under mountain masses, or under low continents, or under the bed of the sea; the visible masses above the sea level are regarded as wholly unconnected with and independent of the invisible masses below; thus the mountains might consist of just so much extraneous stuff cast on the surface of the Earth by passing meteors or the ruined debris of other worlds, whereas we see they must be portions of the Earth's crust derived from the matter lying immediately below them.

It is interesting to make a rough comparison of the relative magnitudes of the greatest horizontal attraction and the greatest vertical diminution of attraction which has been met with hitherto in the Indian Survey. The greatest meridional attraction has been found at Rajpur, in the Dehra Dun Valley, where it amounts to $46^{\prime \prime} \cdot 16$; the greatest attraction on the prime vertical has been found at the Dehra Observatory, where it amounts to about $28^{\prime \prime} \cdot 5$, as shown by the electro-telegraphic

[^4]operations; it may be estimated as $5^{\prime \prime}$ greater at Rajpur; thus the resultant of the two deflections at Rajpur is $57^{\prime \prime}$, and the corresponding horizontal attraction is equal to gravity $\times$ tangent deflection, or 00027 g . The maximum vertical diminution of gravity is* $21^{\prime \prime} \cdot 4$, which corresponds to $\cdot 0005 \mathrm{~g}$, and is thus about twice as great as the horizontal effect.

It is thus evident that the effect of the attraction of mountain masses on the plumb-line, which may be very large in the immediate vicinity of the mountains, will be reduced at a distance in greater proportion than is assigned by an incomplete application of the law of gravitation, because of the deficiency in the density of the strata under the mountains, which has not hitherto been allowed for. Eventually a point must be reached at which the positive attraction of the matter above will be cancelled by the negative attraction of the deficiency below, and then the mountain masses will have no influence on the plumb-line.

We now proceed to examine the differences between the astronomical and the geodetic data, with a view to getting what information we can regarding the magnitudes of the local deflections. The data for the short arcs of this Survey, whether meridional or longitudinal, give a very close representation of the differences of the local deflections at their extremities, because the triangulation is very accurate, and the geodetic errors of arcs not exceeding $2^{\circ}$ or $3^{\circ}$ in length, arising from uncertainty in the elements of the Earth's figure, must be very small; therefore any marked differences between the astronomical and the geodetic amplitudes must be almost wholly due to the deflections at the extreme ends of the arcs. But there are no means of determining the actual amount and direction of the deflection at any single point, because it may be so largely influenced by variations of density underground which are not ascertainable. Thus, the differences between the astronomical and the geodetic latitudes in the preceding tables, pages 778 to 790 , only give the local deflections on the understanding that there is no deflection at the origin.

[^5]Kalianpur was adopted by Everest as the origin, on the assumption that it was free from meridional deflection, the surface of the ground giving no sign of any. To the north, south, and west the same general level prevails for a considerable distance all round; but to the east there is a depression of 200 feet to the plain in which the Sironj base line is situated; thus, while there is apparently no meridional deflection, there may be slight westerly deflection. In section 4 of chapter $x$. of volume 2 it is shown that the observed azimuth at Kalianpur is certainly $1^{\prime \prime} \cdot 1$, and possibly $1^{\prime \prime} \cdot 86$, too great, on the evidence of the astronomical azimuths at sixty-three of the principal stations around Kalianpur; the smaller quantity was applied as a correction to the observed azimuth, to obtain the fundamental azimuth for the final geodetic computations. Now that they are completed it is seen that a still larger correction would have brought the final geodetic azimuths into better accordance with the astronomical. Taking $l^{\prime \prime} \cdot 1$ as the true excess of the astronomical azimuth at Kalianpur, the zenith is deflected $2^{\prime \prime} \cdot 5$ to the east on the prime vertical.

The algebraical mean excess of the whole of the astronomical over the corrected geodetic values of latitude ( $\mathrm{A}-\mathrm{G}_{c}$ ) on the Great Arc, from Punnæ to Kaliana inclusive, which are 54 in number, is $-2^{\prime \prime} \cdot 0$. It will be seen that, of the whole 148 astronomical latitudes available for geodetic investigations, there are 90 cases of negative excess to 58 of positive excess; but, if the latitude of Kalianpur is diminished by $2^{\prime \prime} \cdot 0$, the whole of the geodetic latitudes will be correspondingly diminished, and this will make the number of positive and negative cases almost exactly equal. Thus it is not improbable that the astronomical latitude is $2^{\prime \prime}$ too great, or, in other words, that the plumb-line at Kalianpur is not deflected to the north under the influence of Himalayan attraction, as has been so often suggested, but is in reality deflected to the south. It seems, therefore, that if we subtract $2^{\prime \prime}$ from each of the negative excesses and add $2^{\prime \prime}$ to each of the positive excesses, we shall have a fairly approximate determination of the meridional deflection at each of the latitude stations.

The stations coming under the influence of Himalayan attraction must first be noticed briefly, and afterwards set aside altogether, as they cannot be employed in a geodetic investigation. Proceeding from east to west, the first station, Ramuapur, is situated on the meridian of $80^{\circ} 31^{\prime}$, about 54 miles to the south of the Himalayas, and shows a deflection of $-10^{\prime \prime} \cdot 15$. Then comes the following group of stations on the Great Arc ; Nojli, in the plains, about 30 miles to the south of the Siwaliks, the outer Himalayan range, which rises about 2000 feet above the plains to the south and 1000 feet above the Valley of Dehra Dun to the north, deflection - $12^{\prime \prime} \cdot 86$. Amsot is a peak of the Siwaliks 3140 feet high, deflection $-27^{\prime \prime} \cdot 66$; the Dehra Dun base line station, deflection - $28^{\prime \prime} \cdot 90$; the Dehra Dun Observatory, deflection - $36^{\prime \prime} \cdot 35$; Rajpur, at the foot of the ascent to Mussooree, deflection - 46" 16 ; Mussooree $-35^{\prime \prime} \cdot 12$, and Banog - $31^{\prime \prime} \cdot 54$, stations on the hill ranges where the northerly attraction is diminished because there is attracting matter to the south. Finally, there is a group of stations to the west; Murree, on a mountain to the south of the

Pir Pinjal range, which lies between India and Kashmir, deflection - $18^{\prime \prime} \cdot 00$; and two stations on the northern spurs of the Pir Pinjal, with the wide open valley of Kashmir beyond, and which, therefore, show southerly attraction, namely, Gogipatri, deflection + $5^{\prime \prime} \cdot 03$; and Poshkar, deflection $+16^{\prime \prime} \cdot 79$.

The nearest stations to the Himalayas which may be included in an investigation of the figure of the Earth must now be noticed. They are four in number, as follows:-

| Station. | Latitude. | Longitnde. | $\mathrm{A}-\mathrm{G}_{\text {c }}$. | Distance from Himalayas. |
| :---: | :---: | :---: | :---: | :---: |
| Jarura (Madras meridian) |  | 8031 | $-5^{\prime \prime} 00$ | 80 miles south |
| Kaliána (Great Arc) | 2931 | 7742 | $-5 \cdot 72$ | 52 " " |
| lsanpúr (Rahan meridian) | 3038 | 769 | $-2.75$ | 65 " " |
| Shahpur (Gurhagarh meridian). | $32 \quad 2$ | 758 | +2.00 | 40 " " |

It is to be noticed that the southern foot of the Himalayas descends from latitude $33^{\circ} 50^{\prime}$, on the meridian of $73^{\circ}$, to latitude $29^{\circ}$ on the meridian of $81^{\circ}$, and then trends nearly easterly. Thus the apparent diminution of deflection at Isanpur and reversal at Shahpur, as compared with Jarúra and Kaliána, may be due to the circumstance that the Himalayas descend sume way to their south, though at a very considerable distance to the east; but speculation on this point, is superfluous, as there are so many larger deflections to the south, which cannot possibly be attributed to Himalayan attraction.

Attention must be drawn to group 10 of the Great Arc. It contains one of Lambton's stations, Takalkhera, which Everest rejected, because he calculated that the plumb-line is attracted $5^{\prime \prime}$ northwards at that point by the tablelands of the Mahadeo Hills, which are 20 miles distant and rise about 1600 feet above it. But the group contains four subsequently determined stations whose distances range from 20 to 28 miles to the south of Takalkhera, all which show a still larger amount of northerly attraction. Here, therefore, there must be not only an excess of visible matter above ground in the Mahadeo plateau to the north, but a deficiency of invisible matter underground to the south.

It is only to the operation of underground variations of density that we can look for an explanation of the fact that at Bombay there is apparently large northerly deflection, and at Madras there is large southerly deflection, while there is no visible source of meridional attraction at either place. Whatever the uncertainty about the actual deflection at each place because of the uncertainty at the origin, there is no question that the astronomical amplitude of the arc Bombay-Madras is $16^{\prime \prime} \cdot 01$ less than the (corrected) geodetic amplitude, mainly because of the local deflections at the two places, the difference being at least twenty times greater than can be due to error in the triangulation. There is no apparent cause to which to attribute these gross deflections. Others of a like nature are presented by the latitude observations 5 L 2
at various places, even in the deserts of Rajputana which might be expected to be quite free from local disturbances.

We now proceed to examine the local attractions on longitudinal arcs, as shown in column $\mathrm{A}-\mathrm{G}_{c}$ of Table V., page 799. Being deduced from time observations they are necessarily more affected by errors of observation than the local attractions on meridional arcs, but not to any great extent ; for the maximum circuit error of three arcs is $\cdot 101^{s}$, as shown in Table III., which corresponds to a maximum error of $\cdot 06^{s}$, or less than $1^{\prime \prime}$ of arc, for a single arc. Thus the differences between the observed and the (corrected) geodetic values are a closely approximate indication of the differences between the local attractions on the prime vertical at the extremities of the arcs.

The greatest deflections shown are in the three arcs connecting the station of Dehra Dun, which is situated close under the Himalayas, with Agra to the south, Fyzabad to the east, and Amritsur to the west ; from the mean of the three it is to be inferred that the Himalayan attraction on the prime vertical at Dehra Dun is $28^{\prime \prime} \cdot 5$ easterly. The next greatest deflection is at Peshawur, where it appears from the evidence of the arcs to Mooltan and Amritsur to be about. $14^{\prime \prime} \cdot 4$ westerly, which may well be anticipated from the circumstance that the mountain ranges of Afghanistan lie to the west and a broad open valley to the east. The arc, Jalpaiguri-Fyzabad, shows local deflections, aggregating $15^{\prime \prime} 69$, and there is evidently considerable eastern deflection at Jalpaiguri; but on combining that arc with the arcs to Calcutta and Chittagong it apparently is about $11^{\prime \prime} .8$. Deflections of as much as $10^{\prime \prime}$ to $15^{\prime \prime}$ are met with on arcs connecting stations which might be expected to be almost wholly free from local attraction, as Agra, Mooltan, and Kurrachee; thus showing the action of lidden causes under the surface of the Earth in producing deflections of the plumbline on the prime vertical, such as have already been noticed on the meridian.

Four of the longitude stations are situated on the coasts of the Peninsula, viz., Bombay, Mangalore, Madras, and Waltair. The ocean being on one side and the continent on the other, it is to be supposed that there would be deflection towards the interior of the continent at all four places; but this is only met with at Waltair, where a range of hills 1600 feet higher than the station lies at a distance of only five miles to the west ; at the three other stations there is a more or less wide expanse of plain before hills are reached. The whole of the arcs from the other stations, which are thirteeen in number-six from Bombay, two from Mangalore, four from Madras, and the arc Madras-Mangalore-show deflection towards the ocean and not towards the interior of the continent. The astronomical latitudes in the Southern Peninsula tell the same tale of deflection towards the ocean. All this corroborates the inferences regarding the contraction and condensation of the strata under the bed of the ocean which have already been derived from the pendulum operations.*

[^6]
## Final Results.

Excluding stations obviously affected by the attraction of the great Himalayan and Tibetan masses, we are in possession of 148 astronomical latitudes and 50 longitudinal arcs which are available for an investigation of the mean figure of the Earth. Thus, the question arises as to the manner in which they can be most advantageously employed.

In his 'Geodesy,' Colonel Clarke gives his latest determination of the figure of the Earth, in which he employs 14 of the latitude stations and 6 of the longitudinal arcs of the Indian Surveys, in combination with 35 latitude stations of surveys made in other parts of the world. No attempt will be made here to obtain a new value of the figure of the Earth by employing the whole of the Indian data instead of the small portion used by Colonel Clarke. All that is now aimed at is to set forth the facts to be elicited from the operations in India which will be most serviceable for future mathematical investigations.

The existing data may evidently be treated as they stand, in accordance with either of the various methods which have been employed by Airy, Bessel, Clarke, and others. But in all mathematical treatment of such data it is assumed that the direction of the plumb-line at the astronomical stations is normal, or very closely normal, to the surface, and wherever there is reason to suspect that this may not be the case the station is arbitrarily rejected. Still, as a matter of fact, the paucity of data has frequently led to the employment of stations which are sensibly influenced by local attractions. It is possibly for this reason that some mathematicians have been led to conclude from the data which they have employed that the mean figure of the Earth is not a spheroidal figure, with two axes only, as has been generally accepted on theoretical grounds, but an ellipsoidal figure with three axes, two of which are poles of the equator. This subject has been discussed at length in Archdeacon Pratt's 'Figure of the Earth,' 4th edition, p. 181, with the conclusion that " local attraction appears to supply a source of correction which makes a resort to so peculiar an hypothesis as an ellipsoidal mean figure unnecessary and untenable."

It is obvious, therefore, that, before mathematical treatment can be advantageously commenced, steps should be taken to diminish to the utmost possible extent the local deflections by which the observations are burdened. This cannot be done.by

[^7]minute surveys of the ground and calculations based on the superficial conditions, because no cognizance can be taken of the existing conditions under ground which may be of quite as great or even greater influence in producing deflections of the plumb-line. Thus, in treating meridional arcs, the only possible way is to combine a number of the astronomical stations within a narrow belt of parallel together, and take the mean latitude of the group, and to do the same with the corresponding geodetic determinations, carrying on the process from south to north as far as the data are available. This gives us combinations of data for a single chain of meridional arcs which will be far more valuable for mathematical treatment than the separate individual initial data, because the mean astronomical latitude of a number of points may certainly be assumed to be far more free from deflection than the latitude of any single point.

When this is done for a number of stations extending over a considerable distance from east to west, the objection arises that the procedure binds us to the assumption that the mean figure of the Earth is spheroidal. But all geodetic calculations of the positions of places on the surface of the Earth have been made hitherto on this assumption; for though an ellipsoidal figure has been suggested and calculated, the several calculations which have been made have placed the poles of the equator in such different positions that it does not appear that any person has yet attempted to calculate terrestrial elements of latitude, longitude, and azimuth on the basis of an ellipsoidal figure. We have to begin by assuming something; we have seen that it is impossible to assume any astronomical arcs of amplitude to be free from error caused by local deflections of the plumb-line, which is our only alternative. It seems obvious that an assumption which is in itself highly probable, and which permits of combinations of large numbers of astronomical observations and a consequent large elimination of the errors caused by local deflections, is more likely than any other to lead to exact and definite results.

Combinations of the latitudes will therefore be presently indicated; but, before doing so, it is necessary to fix the most appropriate lengths of the arcs of amplitude to be given by the combinations. The errors caused by the local deflections are independent of the lengths of the arcs; they must produce much greater influence on short than on long arcs ; and therefore it is desirable to fix a relation between the magnitude of the local deflection and the length or amplitude of the arc. Meridional arcs of about $2^{\circ}$ in length appear to have been generally regarded hitherto as giving an adequate number of touching points on the Earth's surface to produce a close representation of the figure; but this length seems to have been fixed without sufficient consideration of the magnitudes of the local deflections involved which are liable to be such a disturbing element in the calculations.

The mathematical relations between errors in arcs of amplitude and errors in the lengths of the Earth's axes are given with sufficient approximation by the following formulæ:-

$$
\begin{aligned}
& d . \Delta \lambda=\frac{1}{a}\left\{d a-2 d b-3(d a-d b) \sin ^{2} \lambda\right\} \Delta \lambda \\
& d . \Delta \mathrm{L}=-\frac{1}{a}\left\{d a+(d a-d b) \sin ^{2} \lambda\right\} \Delta \mathrm{L}
\end{aligned}
$$

If we take the arithmetical mean of the four cases $d a=0, d b=0, d a=d b$, and $d a=-d b$, and put $n$ for the number of degrees in the arc and $1000 p$ for the axial error in feet, and also put $\lambda=21^{\circ}$ the middle latitude of India, we have

$$
\begin{aligned}
& d . \Delta \lambda^{\prime \prime}=0.24 n . p, \\
& d . \Delta \mathrm{L}^{\prime \prime}=0.15 \text { n.p. }
\end{aligned}
$$

Thus for a meridional arc of $2^{\circ}$ and an axial error of 1000 feet, the arc error is only $0^{\prime \prime} \cdot 48$, which is but a small fraction of the error often caused by local deflection. A much larger axial error than 1000 feet must therefore be admitted, or the magnitude of the arc must be so considerably increased that the number of touching points on the Earth's surface will be too small to produce a sufficiently close representation of the surface. Let us put 5000 feet as the limit of axial error, and then we obtain the conditions

$$
n>\frac{d \cdot \Delta \lambda}{1 \cdot 2}, \text { and } n>\frac{d \cdot \Delta \mathrm{~L}}{75},
$$

or in other words, when the arc error is

$$
\begin{aligned}
& 2^{\prime \prime}, n \text { must exceed } 1^{\circ} \cdot 7 \text { in a meridional and } 2^{\circ} .6 \text { in a longitudinal arc. } \\
& \begin{array}{ccccccccc}
3^{\prime \prime}, & " & \# & 2^{\circ} \cdot 5 & \# & " & 4^{\circ} \cdot 0 & \# & " \\
4^{\prime \prime}, & " & " & 3^{\circ} \cdot 3 & \# & " & 5^{\circ} \cdot 3 & " & "
\end{array}
\end{aligned}
$$

These conditions have governed the formation of the following table of combinations of latitudes to produce the final meridional arcs of amplitude. The column $A^{\prime \prime}-A^{\prime}$ gives the differences between the values of the astronomical and the (corrected) geodetic arcs of amplitude, as derived from the data in column $A-G_{c}$; these differences may be assumed to be free from errors caused by local deflections of the plumb-line, and to be mainly due to the errors still latent in Clarke's latest determination of the figure of the Earth.

Combinations of Latitudes and the resulting Arcs of Amplitude.

| Chain of triangles and stations. | Number of stations. | Latitudes. |  | A-G. | $\mathrm{A}-\mathrm{G}_{\mathrm{c}}$. | Arcs of amplitude. |  | $\mathbf{A}^{\prime \prime}-\mathbf{A}^{\prime}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Astronomical. | Geodetic. |  |  | Astronomical. | Geodetic. |  |
| Great Arc. Group 1 . . . . <br> Great Arc. Group 2 . . . . | $\begin{aligned} & 6 \\ & 4 \end{aligned}$ | $\begin{array}{ll} 8 & 12 \\ 9 & 47 \prime 50 \\ 9 & 28 \\ 54: 35 \end{array}$ | $\begin{array}{lll} 8 & 1 \\ \hline & 12 & 45^{\prime \prime} 18 \\ 9 & 28 & 51 \cdot 63 \end{array}$ | $\begin{array}{r} \quad \begin{array}{r} \prime \prime \\ +2.32 \\ +2.72 \end{array} \end{array}$ | $\begin{array}{r} 11 \cdot 65 \\ +\quad \begin{array}{l} 1.02 \end{array} \\ +\quad \end{array}$ | - " " | - . | " |
| Combination I. . . | 10 | 84314.24 | 84311.76 | $+2 \cdot 48$ | $+180$ |  |  |  |
| $\begin{array}{ll} \text { Great Arc. } & \text { Group 3 . . . } \\ \text { Madras. } & \text { Tiruvendiparam } \end{array}$ | $\begin{aligned} & 3 \\ & 1 \end{aligned}$ | $\begin{array}{lrr} 11 & 0 & 3 \cdot 22 \\ 11 & 44 & 43 \cdot 40 \end{array}$ | $\begin{array}{ccc} 11 & 0 & 1 \cdot 79 \\ 11 & 44 & 37 \cdot 53 \end{array}$ | $\begin{aligned} & +1.43 \\ & +5.87 \end{aligned}$ | $\begin{array}{r} +0.72 \\ +5.16 \end{array}$ |  |  |  |
| Combination II. . | 4 | 111113.27 | $111110 \cdot 73$ | $+2.54$ | + 1.83 | 22759.03 | 22758.97 | $+0.03$ |
| Mangalore. Mangalore . . . <br> Madras. Group 1 . . <br> Great Arc. Group 4 . . <br> Mangalore. Group 1 . . <br> Madras. Group 2 . . . .  <br>      | $\begin{aligned} & 1 \\ & 2 \\ & 5 \\ & 2 \\ & 2 \end{aligned}$ | $\begin{array}{llll}12 & 52 & 17 \cdot 76 \\ 13 & 2 & 15 \cdot 27 \\ 13 & 26 & 16.56 \\ 14 & 12 & 16.24 \\ 14 & 14 & 11 \cdot 48\end{array}$ | $\begin{array}{ccc}12 & 52 & 14 \cdot 60 \\ 13 & 2 & 9.31 \\ 13 & 26 & 16.73 \\ 14 & 12 & 19.37 \\ 14 & 14 & 11.85\end{array}$ | $\begin{aligned} & +3.16 \\ & +5.96 \\ & +0.17 \\ & =3.13 \\ & -0.37 \end{aligned}$ | $\begin{array}{r} +\quad 2.46 \\ +5.26 \\ -0.87 \\ -3.82 \\ -1.06 \end{array}$ |  |  |  |
| Combination III. | 12 | $13 \quad 35 \quad 5 \cdot 54$ | 13354.94 | $+0.60$ | $-0.09$ | $22352 \cdot 27$ | $22354 \cdot 21$ | $-1.92$ |
| Great Arc. Group 5 <br> Madras. Group 3 <br> Mangalore. Group 2 <br> Madras. Group 4 <br> Great Arc. Group 6 <br> Mangalor. Group 3 <br> Great Arc. Group 7 <br> Madras. Group 5 | $\begin{aligned} & 6 \\ & 2 \\ & 2 \\ & 2 \\ & 2 \\ & 5 \\ & 2 \\ & 5 \\ & 2 \end{aligned}$ | $\begin{array}{llll}14 & 56 & 24 \cdot 50 \\ 15 & 15 & 13.21 \\ 15 & 20 & 21.47 \\ 16 & 10 & 56 \cdot 59 \\ 16 & 11 & 52 \cdot 83 \\ 16 & 35 & 59.96 \\ 17 & 8 & 11 \cdot 09 \\ 17 & 21 & 57 \cdot 54\end{array}$ | $\begin{array}{llr}14 & 56 & 22 \cdot 16 \\ 15 & 15 & 16 \cdot 50 \\ 15 & 20 & 23 \cdot 07 \\ 16 & 10 & 58 \cdot 29 \\ 16 & 11 & 58 \cdot 05 \\ 16 & 36 & 0.35 \\ 17 & 8 & 14.73 \\ 17 & 22 & 4.57\end{array}$ | $\begin{aligned} & +2.34 \\ & =3.29 \\ & =1.60 \\ & =1.70 \\ & =5.22 \\ & =0.39 \\ & =3.64 \\ & -7.03 \end{aligned}$ | $\begin{aligned} & +1.67 \\ & =3.95 \\ & -2.26 \\ & =2.34 \\ & =5.86 \\ & =1.02 \\ & =4.24 \\ & -7.62 \end{aligned}$ |  |  |  |
| Combination IV. | 26 | $\begin{array}{llll}16 & 4 & 8 \cdot 62\end{array}$ | $16 \quad 4 \quad 10 \cdot 86$ | $-2.24$ | $-2.88$ | 2293.08 | 229 | $-2.79$ |

General j. t. Walker on india's contribution to geodesy

Combinations of Latitudes and the resulting Arcs of Amplitude-(continued).


Combinations of Latitudes and the resulting Arcs of Amplitude-(continued).

| Chain of triangles and stations. | $\begin{gathered} \text { Number } \\ \text { of } \\ \text { stations. } \end{gathered}$ | Latitudes. |  | A-G. | $\mathrm{A}-\mathrm{G}_{\mathrm{c}}$. | Arcs of amplitade. |  | $A^{\prime \prime}-A^{\prime}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Astronomical. | Geodetic. |  |  | Astronomical. | Geodetic. |  |
| Calcutta-Karachi longitadinal | 7 |  | $\begin{array}{llll}24 & 1 & 4 \prime 87\end{array}$ | +0.98 | +0.97 | - ، ${ }^{\prime}$ | - ، " | " |
| Great Arc. Origin. Kalianpar | 1 | ${ }^{24} 471110$ | $\begin{array}{llll}24 & 7 & 11 & 10\end{array}$ |  |  |  |  |  |
| Bombay. Group 1. . . . | 3 | $\begin{array}{llll}24 & 19 & 51 \cdot 48\end{array}$ | ${ }^{24} 4191958 \cdot 82$ | $-7.34$ | -7:31 |  |  |  |
| Madras. Group 11. | 2 | $\begin{array}{lll}24 & 21 & 3\end{array}$ | $\begin{array}{lll}24 & 21 & 21 \\ 24 & 2 & 37\end{array}$ | +1.09 | +1.12 |  |  |  |
| Mangalore. Gronp 7. | 2 | 24 <br> 24 <br> 25 <br> 17 1732 | 24 24 25 151971 | -2.39 | $-2 \cdot 35$ |  |  |  |
| Great Arc. Group 11. | 2 | 252124.02 | 25 21 21 $21 \cdot 21$ | +2.81 | +296 |  |  |  |
| Mangalore. Group 8. | $\stackrel{2}{2}$ | 25 <br> 41 <br> 254 <br> 187 | ${ }^{25} 54137 \cdot 11$ | -2.24 | -2.04 |  |  |  |
| Madras. Group 12. | 3 | $255135 \cdot 16$ | $255130 \cdot 25$ | +4.91 | +5.21 |  |  |  |
| Bombay. Group 2 . | 3 | 261038.68 | ${ }_{26} 261039 \cdot 22$ | $-0.54$ | -0.27 |  |  |  |
| Mangalore. Group 9 . | 3 | 264041.71 | $264042 \cdot 59$ | -0.88 | -0.53 |  |  |  |
| Combination VII. . | 28 | $25 \quad 2 \quad 12.59$ | $25 \quad 212.81$ | -0.22 | -0.11 | 2285959 | $22257 \cdot 44$ | +2.49 |
| Great Arc. Group 12. | 2 | $\begin{array}{lll}27 & 3 & 17.51 \\ 27 & 7 & 45 \cdot 40\end{array}$ | $\begin{array}{lll}27 & 3 & 22 \cdot 92 \\ 27 & 7 & 42 \cdot 81\end{array}$ | -5.41 +2.59 | -5.00 |  |  |  |
| Madras. Group 13. | 2 | $\begin{array}{llll}27 & 7 & 45 \cdot 40\end{array}$ | $\begin{array}{llll}27 & 7 & 42 \cdot 81\end{array}$ | +2.59 | +3.00 |  |  |  |
| Bombay. Group 3 | 2 | $273458 \cdot 52$ | $2734 \begin{aligned} & 27 \\ & 27 \\ & 50\end{aligned}$ | +3.22 | +372 |  |  |  |
| Madras. Jarúra. | 1 | $275950 \cdot 22$ | $275955 \cdot 78$ | -5.56 | --5.00 |  |  |  |
| Mangalore. Group 10. | 3 | $\begin{array}{llll}28 & 6 & 2 & 2.78\end{array}$ | $\begin{array}{llll}28 & 6 & 6 & 2 \cdot 93\end{array}$ | $-0 \cdot 15$ | $+0 \cdot 43$ |  |  |  |
| Great Arc. Group 13. | 2 | $28 \quad 17 \quad 25 \cdot 90$ | $281728 \cdot 63$ | $-2.73$ | -2.11 |  |  |  |
| Combination VIII . | 12 | $2742 \quad 443$ | $2742 \quad 5 \cdot 32$ | -0.89 | -0:37 | 2395184 | $23952 \cdot 51$ | -0.26 |
| Bombay. Group 4 | 3 | 285552.01 | $\begin{array}{ll}28 & 55 \\ 51 & 12\end{array}$ | +0.89 | +1.29 |  |  |  |
| Mangalore. Group 11. | 2 | $29 \quad 28 \quad 16.94$ | $\begin{array}{ll}29 & 28 \\ 17 & 176\end{array}$ | $-0.52$ | +0.32 |  |  |  |
| Great Arc. Kaliána . | 1 | 293047.98 | 293054.54 | $-6.56$ | $-5.72$ |  |  |  |
| Bombay. Group 5 | 2 | $30 \quad 2227.93$ | $302230 \cdot 13$ | -2 20 | -1.18 |  |  |  |
| Mangalore. Group 12. | 2 | 30 30 $13 \cdot 89$ | $303017 \cdot 26$ | $-3.37$ | $-2 \cdot 33$ |  |  |  |
| Mangalore. Group 13. . . . . | 3 | $\begin{array}{ll}31 & 39 \\ 4\end{array}$ | 31 39 $2 \cdot 15$ | +1.91 | +3.20 |  |  |  |
| Combination 1X. | 13 | $\begin{array}{llll}30 & 9 & 2\end{array} 59$ | $\begin{array}{llll}30 & 9 & 3.39\end{array}$ | -0.80 | +0.10 | 2 2658.16 | 22658.07 | +0.47 |

From the preceding combinations we obtain the following table of what appear to be the most trustworthy results at present derivable from the operations of the Indian Survey:-

Resulting Meridional Arcs.

| Combinations. | Astronomical latitudes. |  |  | Astronomic | amplitudes. | Arc-lengths in feet. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I. | $\begin{array}{llll}8 & 4 & 43 & 14: 24\end{array}$ |  |  |  |  | 894,947•2 |
| II. |  |  |  |  |  |  |
|  | $\begin{array}{llll}11 & 11 & 13 \cdot 27\end{array}$ |  |  | $\begin{array}{lll} 2 & 23 & 52 \cdot 27 \end{array}$ |  | 870,416.6 |
| III. | $\begin{array}{lll}13 & 35 & 5 \cdot 54\end{array}$ |  |  |  |  |  |
| IV. |  |  |  | $\begin{array}{llll}2 & 29 & 3\end{array}$ |  | 902,015•2 |
|  | $\begin{array}{llll}16 & 4 & 8.62\end{array}$ |  |  | $\begin{array}{llll}2 & 57 & 779\end{array}$ |  | 1,072,166.9 |
| V. | $\begin{array}{lll}19 & 1 & 16 \cdot 41\end{array}$ |  |  |  |  |  |
| VI. | 22 39. 13.00 |  |  | $\begin{array}{llll}3 & 37 & 56.59\end{array}$ |  | 1,319,028.9 |
|  |  |  |  | $\begin{array}{llll}2 & 22 & 59.59\end{array}$ |  | 865,705•7 |
| VII. | $\begin{array}{lll}25 & 2 & 12 \cdot 59\end{array}$ |  |  |  |  |  |
|  | $\begin{array}{llll}27 & 42 & 4 \cdot 43\end{array}$ |  |  | $\begin{array}{llll}2 & 39 & 51.84\end{array}$ |  | 968,481-6 |
|  |  |  |  | $\begin{array}{llll}2 & 26 & 58\end{array}$ |  | 890,617.5 |
| IX. |  | 9 | $2 \cdot 59$ |  |  |  |

The arc-lengths in feet are deduced from the original geodetic amplitudes which are converted into feet by the usual formulæ.

Fur the Longitudinal Arcs the treatment is simpler. As we assume the figure of the Earth to be spheroidal, and, therefore, the length of an arc on any given parallel of latitude to be exactly proportional to its amplitude, we have simply to form the longest arcs on the most convenient parallels of latitude which we can obtain from combinations of the given arcs. The whole of the ares having been made consistent by the mathematical treatment which has been described, it does not matter which of them are employed in forming a combination; but it will be convenient to take those which are nearest to the selected parallels. The combinations have the immense advantage of being free from all errors arising from local deflections of the plunbline, excepting those which occur at the extreme eastern and western stations of the arcs of parallel, those occurring at the intermediate stations becoming eliminated. Thus the magnitudes of the deflections which remain and cannot be eliminated will be reduced from large and significant fractions of small arcs to comparatively small and insignificant fractions of large arcs; at least this will happen in three out of the four cases to be now brought forward, the exception being the arc between Madras and Mangalore, which is necessarily a short one, the length being curtailed by the Indian Ocean.

The following table gives the adopted longitudinal arcs and their combinations. It is arranged to show the excesses of the astronomical amplitudes over both the
original geodetic values and those values corrected to accord with the latest determined lengths of the Earth's axes, as a preliminary to obtaining estimates of the most probable values of the magnitudes of the local deflections of the plumb-line, in accordance with what has been previously done for the meridional arcs. It will be seen that what was of comparatively little importance for the meridional determinations is of considerable importance for the longitudinal.

Longitudinal Arcs and their Combinations.

| Names of arcs and combinations. | Amplitudes. |  | A-G. | $A-G_{c}$. |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | A stronomical. | Geodetic. |  | In time. | In arc. |
| First Arc.-Madras-Mangalore. Mean latitude $12^{\circ} 58^{\prime} 8^{\prime \prime} \cdot 95$. <br>  |  |  |  |  |  |
| Second Aire-Moulmein-Bombay Combination. Mean latitude $17^{\circ} 41^{\prime} 52^{\prime \prime} \cdot 99$. |  |  |  |  |  |
| Moulmein-Akyab | 1854.985 | $1855 \cdot 432$ | -0.447 | $-0.260$ | $-3.90$ |
| Akgab-Calcutta | $18 \quad 9.386$ | $18 \quad 9 \cdot 431$ | -0.045 | +0.137 | + 2.06 |
| Calcatta-Waltair | $\begin{array}{lll}20 & 9 \cdot 178\end{array}$ | $\begin{array}{ll}20 & 9.684\end{array}$ | -0.506 | -0.305 | - 4.58 |
| Waltair-Bolaram | $19 \quad 11 \cdot 489$ | ${ }_{19} 111482$ | +0.007 | +0.196 | + 2.94 |
| Bolarum-Bombay | 2248.801 | $2249 \cdot 479$ | -0.678 | $-0.452$ | $-6.78$ |
| II. Moulmein-Bombay | 9913839 | 9915.508 | -1.669 | -0.684 | -10.26 |
| Thirl Arc.-Chittagong-Kurrachee Combination. Mean latitude $23^{\circ} 35^{\prime} 40^{\prime \prime} \cdot 99$, |  |  |  |  |  |
| Clittagong-Calcutta | 1355.138 | $1355 \cdot 195$ | -0.057 | +0.084 | $+1.26$ |
| Calcutta-Jubbulpore | $3337 \cdot 695$ | $3337 \cdot 744$ | $-0.049$ | +0.292 | + 4.38 |
| Jubbulpore-Deesa | $31 \quad 3 \cdot 410$ | $31 \quad 3 \cdot 851$ | $-0.441$ | -0.125 | $-1.87$ |
| Deesa-Kurrachee | $2040 \cdot 559$ | $2040 \cdot 775$ | -0.216 | -0.004 | - 0.06 |
| III. Chittagong-Kurrachee | 99168802 | 9917565 | $-0.763$ | + $6 \cdot 247$ | +3.71 |
| Fourtı Arc.-Jalpaiguri-Quetta Combination. Mean latitude $28^{\circ} 21^{\prime \prime} 37^{\prime \prime} \cdot 34$. |  |  |  |  |  |
| Jalpaiguri-Fyzabad Fyzabad-Agra | 26 1628.988 16 | $\begin{array}{ll}26 & 24.308 \\ 16 & 28 \cdot 417\end{array}$ | -1.320 -0.410 | -1.046 -0.239 | -1569 -3.59 |
| Agra-Mooltan | 2619.020 | 2618300 | +0.720 | +0.997 | +14.96 |
| Mooltan-Quetta | 1743528 | 1743714 | $-0.186$ | +0.002 | + 0.03 |
| IV. Jalpaiguri-Quetta | 8653.543 | S6 54.739 | $-1 \cdot 196$ | -0.286 | $-4.29$ |

Thus we obtain the following combination data for longitudinal ares, as we already have done for meridional arcs:-

Resulting Longitudinal Arce.

| Number of Arc. | Mean latitude. | Astronomical amplitude. | Arc-lengths, in feet. |
| :---: | :---: | :---: | :---: |
| I. | $\stackrel{\circ}{12} 58888$ |  | 1,923,108 |
| II. | 174152.99 | $244827 \cdot 59$ | 8,635,432 |
| III. | $233540 \cdot 99$ | $244912 \cdot 03$ | 8,311,387 |
| IV. | $282137 \cdot 34$ | $214323 \cdot 15$ | 6,987,444 |

The first arc is too short to be employed in a geodetic investigation, because of the magnitude of the error in the astronomical amplitude, apparently caused by local deflections of the plumb-line towards the strata under the bed of the ocean at each extremity of the arc ; the rule which has been adopted to govern the relations between the amplitude of an arc and the magnitude of the deflections requires that for such an error as we have here, apparently $6^{\prime \prime} \cdot 15$, the length of the arc should be at least half as much again as it is.
The second are is good, though probably influenced to some extent by deflection towards the strata under the ocean at Bombay; but this is of little significance in an arc of so great length.

The third arc is best of all, as it is apparently least affected by local deflections of the plumb-line, and is of considerable length.
The fourth are is not so good as the smallness of the sum of the local deflections suggests; it is burdened with considerable local deflection at Jalpaiguri, its eastern extremity, and possibly also at Quetta, its western extremity, which is situated on an elevated plateau 5500 feet high, and is surrounded by mountains which rise to a much higher altitude on the east than on the west; moreover, the geodetic connection between Mooltan and Quetta rests on a long single chain of secondary triangles, and is, therefore, much weaker than that of any of the other longitude stations, all of which are more or less immediately connected with the principal triangulation.

Thus the results which may be put forward as essentially representing India's contribution to geodesy consist of eight meridional arcs extending from latitude $8^{\circ} 43^{\prime} 14^{\prime \prime}$ to latitude $30^{\circ} 9^{\prime} 3^{\prime \prime}$, and resting-not as usual on latitude observations taken at one more station than the number of arcs, but-on latitude observations taken at 148 stations, giving an average of over 16 stations for each of the funda-
mental latitudes; there are also two excellent longitudinal arcs on the parallels of $17^{\circ} 42^{\prime}$ and $23^{\circ} 36^{\prime}$ each exceeding $24^{\circ}$ in amplitude and $8,300,000$ feet in length.

In all mathematical investigations of the figure of the Earth which have yet been made, individual results have invariably been employed and never any combinations of results for the purpose of diminishing errors caused by local deflections of the plumb-line. This has almost certainly arisen from the circumstance that the astronomical data have been too few to be otherwise employed. But the Indian Survey furnishes abundant data, far exceeding what has ever been put forward hitherto by any other survey; and these data not only indicate the general presence of large local disturbances of the plumb-line and make it more prominent than appears to have been generally recognized hitherto, but they admit of and suggest, they almost necessitate, a system of combination of results which will have the happy result of greatly diminishing the influence of deflections of the plumb-line in future investigations of the Earth's figure.

The eight meridional and the two longest longitudinal arcs here given, are believed to be the most valuable contribution to geodetic science that has yet been made. They are the result of operations which have been carried on in India during a period of over ninety years, with more or less vigour at different times, but always with the cordial support and approval of the Government of India.

## Description of Plate.

[^8]
©EL H.R.THUILLIER, C. I. E., R.E. Surveque-G
October 1893.


## GAnACHAns

$\because$

$$
\because \quad \because
$$


[^0]:    * "Geodeny," by Colonel A. R. Clarke, C.B., Clarendon Press Series. Oxford, 1880.

[^1]:    * Trojghton and Simms' 36 -inch theodolite and their two 18 -inch theodolites were landed in India in 1830. The larger instrument was excellently divided by hand by Mr. William Simms. The two smaller instruments are alleged to have been divided upon Troughton's first dividing engine, but the marked differences in their performances, and the existence of a considerable periodic error in No. 1 , from which No. 2 is free, indicate differences in the method of graduating. The original dividing engine was found to be faulty, and was superseded by the celebrated dividing engine which was completed in the year 1842, and has afterwards been employed in graduating all the large circles manufactured by this firm. Their two 24 -inch theodolites which appear in the list on $p$. 751 , were received in India in 1848, and were graduated by the new machine. In the same year Barrow's two 24 -inch theodolites were received; they are remarkably well graduated instruments; the angles measured with No. 2 in the North-West Quadrilateral show the smallest probable errors of any of the observed angles; they were, however, measured under exceptionally favourable atmospheric conditions. Wavgh's 24 -inch theodolites were put together in Calcutta in 1846 from the parts of other instruments; most of the other 18-inch and 15 -inch theodolites were similarly constructed, and they all date back to some period anterior to 1830.

[^2]:    * The towers consisted at first of a solid masonry pillar, on which the instrument was placed, surrounded by a platform of sun-dried bricks and earth for the observer and his equipment; after deflection was noticed the pillars were built hollow with a vault below, and the markstone was placed at the basement, with a passage leading to it.

[^3]:    * Approximate values

[^4]:    *See p. 29 of vol. 5, which gives an account of the Indian Pendulum Operations.

[^5]:    * It must here be pointed out that there is a slight uncertainty in this vibration number, because of the inadequacy of the data for its calculation. In reducing pendulum observations to the sea level, it is necessary to apply two corrections, one for the height of the station, which is always positive, the other for the ratio of the density of the elevated mass to the mean density of the earth, which is always negative. The formula for the double correction which has been employed, for the reasons set forth in chapter is. of volume 5 , is

    $$
    \delta N=\frac{N_{0}}{a} . H\left(1-1 \cdot 005 \frac{3}{4} \frac{\sigma}{\rho}\right)
    $$

    where $N_{0}$ is the equatorial vibration number of the pendulum, $a$ the Earth's semi-major axis, $H$ the height above sea level, $\sigma$ the density of the clevated mass, and $\rho$ the mean density of the Earth. Now the values of both $\sigma$ and $\rho$ are only known approximately. Here it is assumed that $\sigma=\frac{1}{2} \rho$. Professor Young, who first pointed out the necessity for making an allowance for the density of the elevated mass, assumed that $\sigma: \rho:: 2 \cdot 5: 5 \cdot 5$. Employing his figures, the deíciency at More, the Himalayan tableland station, would be reduced from $21^{\prime \prime} \cdot 44$ to $19^{\prime \prime} \cdot 74$. On the other hand, if the density ratio is larger than $\mathbf{l : 2}$, as is quite possible, the deficiency in the vibration number will be increased. On any reasonable assumption whatever there will always be a marked deficiency in the density of the strata which lie underneath the elevated mass.
    MDCCCXCV.-A.

[^6]:    * This contraction is suggestive of considerable depth below the sea level in the bed of the ocean,

[^7]:    near the coast line. The Admiralty charts give the following details on this subject:-At Bombay there is a belt of water of a depth of about 250 feet extending 150 miles westwards, beyond which are sonndings of 5,000 to 10,000 feet. At Mangalore a belt of much the same depth extends 50 miles westwards, beyond which are soundings of 6,000 and 7,000 feet. At Madras there are soundings of 5,000 to 10,000 feet within 30 miles of the coast line. At Waltair there is a sounding of 8,000 feet within 50 miles of the coast line. The charts show a line of demarcation between deep and shallow water on the west coast, but there is no such line on the east coast. The shallows may very possibly be recent deposits on ancient contracted beds.

[^8]:    The accompanying Index Chart of the Great Trigonometrical Survey of India shows the whole of the Principal Triangalation, the Stations at which Astronomical Latitudes have been observed, and the Electro-Telegraphic Longitudinal Ares; also the Pendulum Stations. It also shows the Azimuth Stations, the lines of the Spirit Levelling Operations, and the Secondary Triangulation to the Himalayan Mountains and various points external to the Principal ; but it omits for the sake of clearness all the large amount of internal Secondary Triangulation which has been executed.

